1\. (a)

critical point: An interior point of the domain of a function f where f' is zero or undefined is a critical point of f.
(10%)

(b)

Mean Value Thm: Suppose that $y=f(x)$ is continuous on a closed interval $[a,b]$ and differentiable on the interval's interior (a,b). Then there is at least on point c in (a,b) at which $\frac{f(b)-f(a)}{b-a}=f'(c)$
(15%)

2\. $y=f(x)=\frac{8x}{x^2+4}$

$f'(x)=\frac{8(x^2+4)-8x(2x)}{(x^2+4)^2}=\frac{32-8x^2}{(x^2+4)^2}$ 令 $=0 \Rightarrow 8x^2=32 \Rightarrow x=\pm 2$ 時 $f'(x)=0$. (5%)

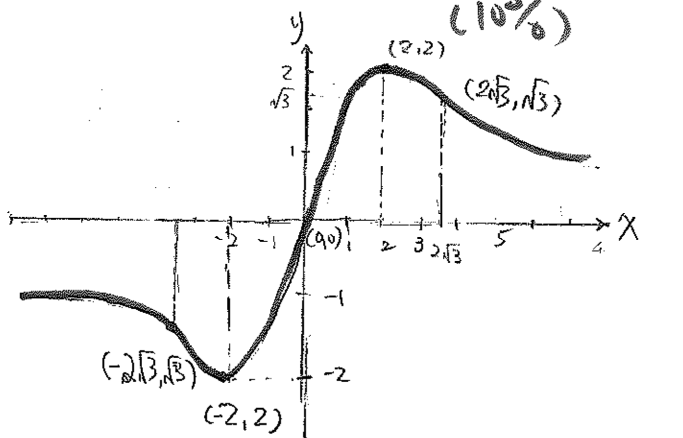
$f''(x)=\frac{(x^2+4)^2(-16x)-2(x^2+4)\cdot(2x)(32-8x^2)}{(x^2+4)^4}$ 令 $=0 \Rightarrow (x^2+4)^{\cancel{2}}(-16x)=\cancel{2}(\cancel{x^2+4})(2x)(32-8x^2)$ $(4-x^2)$ (5%)

$\Rightarrow -x(x^2+4)=2x(4-x^2) \Rightarrow x(-x^2-4+2x^2-8)=0 \Rightarrow x(x^2-12)=0 \Rightarrow x=0,\pm 2\sqrt{3}$ 時 $f''(x)=0$

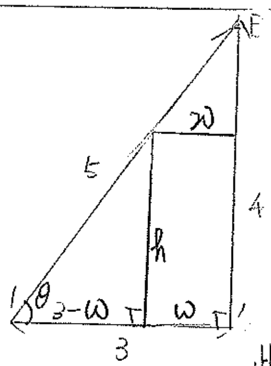
~~(15%)~~ (5%)

x		$-2\sqrt{3}$		-2		0		2		$2\sqrt{3}$	
y		$-\sqrt{3}$		-2		0		2		$\sqrt{3}$	
$f'(x)$	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$
$f''(x)$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
graph	↘		↳		↗		↗		↘		↘

(10%)



3\.



$\because \tan\theta=\frac{h}{3-w}=\frac{4}{3} \Rightarrow h=4-\frac{4}{3}w$ (5)

$A(w)=w\cdot(4-\frac{4}{3}w)$ (5)

$=4w-\frac{4}{3}w^2$

$A'(w)=4-2\cdot\frac{4}{3}w$ 令 $=0$ (5)

$\Rightarrow 2\cdot\frac{4}{3}w=4 \Rightarrow w=\frac{3}{2}$ (4)

此時 $h=4-\frac{4}{3}\cdot\frac{3}{2}=2$, $A(w)=3$

在 $h=2$, $w=\frac{3}{2}$ 時, 有最大面積 3 (5)

另: $w=3-\frac{3}{4}h$

$A(h)=h\cdot(3-\frac{3}{4}h)$

$=3h-\frac{3}{4}h^2$

$A'(h)=3-\frac{3}{2}h$ 令 $=0 \Rightarrow h=2$

此時 $w=3-\frac{3}{4}\times 2=\frac{3}{2}$, $A(h)=3$

4\. $f(x)=x^4+x-3$

$f'(x)=4x^3+1$

$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$

$=x_n-\frac{x_n^4+x_n-3}{4x_n^3+1}$

(15)

left-band zero:

$x_0=-1$

$x_1=-1-\frac{1-1-3}{-4+1}=-1-\frac{3}{3}=-2$

$x_2=-2-\frac{(-2)^4-2-3}{4(-2)^3+1}$

$=-2-\frac{16-5}{-32+1}=-2+\frac{11}{31}$

$=\frac{-51}{31}$ ※

(5)

right-band zero.

$x_0=1$

$x_1=1-\frac{1+1-3}{4+1}=1+\frac{1}{5}=\frac{6}{5}$

$x_2=\frac{6}{5}-\frac{(\frac{6}{5})^4+(\frac{6}{5})-3}{4\cdot(\frac{6}{5})^3+1}\approx \underline{1.165}$

(5)