THOMAS' CALCULUS (12/E)

7.5 Indeterminate Forms and L'Hopital's Rule

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
授課教師: 吳漢銘 (國立臺北大學統計學系 副教授)
教學網站: http://www.hmwu.idv.tw

系級: ________ 學號: ________ 姓名: ________

1 Indeterminate Form 0/0

1.1 If the continuous functions $f(x)$ and $g(x)$ are both ________ at $x = a$, then ________ cannot be found by substituting $x = a$.

1.2 The substitution produces ________, a meaningless expression, which we cannot evaluate. We use ________ as a notation for an expression known as an ________.

1.3 *Theorem: L'Hopital's Rule (First Form)*

Suppose that ________, that $f'(a)$ and $g'(a)$ exist, and that $g'(a) \neq 0$. Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \underline{\qquad\qquad}.$$

Proof:

1.4 *Theorem: L'Hopital's Rule (Stronger Form)*

Suppose that ________________, that f and g are differentiable on an open interval I containing a, and that $g'(x) \neq 0$ on I if $x \neq a$. Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \underline{\hspace{3cm}},$$

assuming that the limit on the right side exists.

1.5 *Using L'Hopital's Rule*

To find $\lim_{x\to a} \frac{f(x)}{g(x)}$ by L'Hopital's Rule,

(a) continue to differentiate f and g, so long as we still get the form ________ at $x = a$.

(b) But as soon as one or the other of these derivatives is different from ________ at $x = a$ we stop differentiating.

(c) L'Hopital's Rule does not apply when either the ____________ or ______________ has a finite __________ limit.

Ex. 1 .... (`example1, p397`)

(a) $\lim_{x\to 0} \frac{3x - \sin x}{x} =$

(b) $\lim_{x\to 0} \frac{\sqrt{1+x} - 1}{x} =$

(c) $\lim_{x\to 0} \frac{\sqrt{1+x} - 1 - x/2}{x^2} =$

(d) $\lim_{x\to 0} \frac{x - \sin x}{x^3} =$

Ex. 2 .... (example2, p398)

Find $\lim_{x \to 0} \frac{1 - \cos x}{x + x^2}$.

sol:

Ex. 3 .... (example3, p398)

(a) $\lim_{x \to 0^+} \frac{\sin x}{x^2} =$

(b) $\lim_{x \to 0^-} \frac{\sin x}{x^2} =$

2 Indeterminate Form $\infty/\infty, \infty \cdot 0, \infty - \infty$

2.1 L'Hopital's Rule applies to the indeterminate form ________.

2.2 If ____________ and ____________ as $x \to a$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \underline{\hspace{3cm}}$$

provided the limit on the right exists.

2.3 In the notation $x \to a$ may be either ________ or ________.

2.4 Moreover $x \to a$ may be replaced by the one-sided limits ________ or ________.

Ex. 4 .... (example4, p398)

(a) $\lim_{x \to \pi/2} \frac{\sec x}{1 + \tan x} =$

(b) $\lim_{x \to \infty} \frac{\ln x}{2\sqrt{x}} =$

(c) $\lim_{x\to\infty} \frac{e^x}{x^2} =$

✎ Ex. 5 .... (example5, p399)

(a) Find $\lim_{x\to\infty} (x \sin \frac{1}{x})$.

(b) Find $\lim_{x\to 0^+} (\sqrt{x} \ln x)$.

sol:

✎ Ex. 6 .... (example6, p399)

Find $\lim_{x\to 0} (\frac{1}{\sin x} - \frac{1}{x})$.

sol:

3 Indeterminate Powers

3.1 If $\lim_{x\to a} \ln f(x) = L$, then $\lim_{x\to a} f(x) =$ __________________. Here a may be either finite or infinite.

3.2 *Theorem: Cauchy's Mean Value Theorem*

Suppose functions f and g are continuous on $[a, b]$ and differentiable throughout (a, b) and also suppose $g'(x) \neq 0$ throughout (a, b). Then there exists a number c in (a, b) at which

$$\frac{f'(c)}{g'(c)} = \underline{\hspace{3cm}}.$$

Ex. 7 . . . . (`example7, p400`)

Apply l'Hôpital's Rule to show that $\lim_{x\to 0^+}(1+x)^{1/x} = e$

sol:

Ex. 8 . . . . (`example8, p400`)

Find $\lim\limits_{x\to\infty} x^{1/x}$.

sol:

實習課練習 (EXERCISE 7.5)

Use L'Hôpital Rule to find the limits.

5. $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.

8. $\lim_{x\to -5} \frac{x^2-25}{x+5}$.

14. $\lim_{t\to 0} \frac{\sin 5t}{2t}$.

20. $\lim_{x\to 1} \frac{x-1}{\ln x - \sin \pi x}$.

27. $\lim_{\theta\to 0} \frac{3^{\sin\theta}-1}{\theta}$.

29. $\lim_{x\to 0} \frac{x2^x}{2^x-1}$.

34. $\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x}$.

41. $\lim_{x\to 1^+} (\frac{1}{x-1} - \frac{1}{\ln x})$.

46. $\lim_{x\to\infty} x^2 e^{-x}$.

48. $\lim_{x\to 0} \frac{(e^x-1)^2}{x\sin x}$.

53. $\lim_{x\to\infty} (\ln x)^{1/x}$.

58. $\lim_{x\to 0} (e^x+x)^{1/x}$.

59. $\lim_{x\to 0^+} x^x$.

60. $\lim_{x\to 0^+} (1+\frac{1}{x})^x$.

62. $\lim_{x\to\infty} (\frac{x^2+1}{x+2})^{1/x}$.

73. $\lim_{x\to\infty} \frac{e^{x^2}}{xe^x}$.