THOMAS' CALCULUS (12/E)

8.1 Integration by Parts

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
授課教師: 吳漢銘 (國立臺北大學統計學系 副教授)
教學網站: http://www.hmwu.idv.tw

系級: _____ 學號: _____ 姓名: _____

1 Basic Integration Formulas

1.1 Substitution Rule: $u = g(x)$, _____

1.2 Basic formulas:

(1) $\int du =$ _____

(2) $\int k\ du =$ _____

(3) $\int (du + dv) =$ _____

(4) $\int u^n\ du =$ _____, $n \neq -1$

(5) $\int \frac{du}{u} =$ _____

(6) $\int \sin u\ du =$ _____

(7) $\int \sec^2 u\ du =$ _____

(8) $\int \csc^2 u\ du =$ _____

(9) $\int \sec u \tan u\ du =$ _____

(10) $\int \csc u \cot u\ du =$ _____

(11) $\int \tan u\ du =$ _____ = _____

(12) $\int \cot u \, du =$ _____ $=$ _____

(13) $\int e^u \, du =$ _____

(14) $\int a^u \, du =$ _____

(15) $\int \frac{du}{\sqrt{a^2 - u^2}} =$ _____

(16) $\int \frac{du}{a^2 + u^2} =$ _____

(17) $\int \frac{du}{u\sqrt{u^2 - a^2}} =$ _____

2 Product Rule in Integral Form

2.1 Integration by parts: if f and g are differentiable functions of x,

(a) The Product Rule: $\frac{d}{dx}[f(x)g(x)] =$ _____.

(b) In term of indefinite integrals:

$$\int \frac{d}{dx}[f(x)g(x)] \, dx = \text{\_\_\_\_\_}.$$

(c) Integration by parts:

_____.

2.2 Let $u = f(x)$ and $v = g(x)$, then $du = f'(x) \, dx$ and $dv = g'(x) \, dx$. Using substitution rule, the integration by parts formula becomes _____.

2.3 Note:

(a) The goal of integration by parts is to go from an integral _____ that we don't see how to evaluate to an integral _____ that we can evaluate.

(b) The integration by parts does not always work.

2.4 Integration by Parts Formula for Definite Integrals

$$\int_a^b f(x)g'(x) \, dx = \text{\_\_\_\_\_}$$

✎ Ex. **1** ..... (example1, p437)

Find $\int x \cos x \; dx$

sol:

✎ Ex. **2** ..... (example2, p3)563

Find $\int \ln x \; dx$

sol:

✎ Ex. **3** ..... (example3, p437)

Find $\int x^2 e^x \; dx$

sol:

✎ Ex. **4** ..... (example4, p438)

Find $\int e^x \cos x \ dx$

sol:

3 Tabular Integration

✎ Ex. 5 ..... (example7, p440)

Evaluate $\int x^2 e^x \ dx$.

sol:

✎ Ex. **6** ..... (example8, p441)

Evaluate $\int x^3 \sin x \; dx$

sol:

實習課練習 (EXERCISE 8.1)

1. $\int x \sin \frac{x}{2}\, dx$

6. $\int_1^e x^3 \ln x\, dx$

16. $\int p^4 e^{-p}\, dp$

19. $\int x^5 e^x\, dx$

21. $\int e^\theta \sin\theta\, d\theta$

25. $\int e^{\sqrt{3s+9}}\, ds$

28. $\int \ln(x + x^2)\, dx$

36. $\int \frac{(\ln x)^3}{x}\, dx$

44. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

46. $\int \sqrt{x} e^{\sqrt{x}}\, dx$

62. Establish the reduction formula: $\int x^n \sin x\, dx = -x^n \cos x + n \int x^{n-1} \cos x\, dx$.