THOMAS' CALCULUS (12/E)

12.3 The Dot Product

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1 Angle Between Vectors

1.1 *Definition*

The dot product $\vec{u} \cdot \vec{v}$ of vectors $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ is

$$\vec{u} \cdot \vec{v} = \_\_\_\_\_$$

1.2 *Theorem 1*

The angle θ between two nonzero vectors $\vec{u} =< u_1, u_2, u_3 >$ and $\vec{v} =< v_1, v_2, v_3 >$ is given by

$$\theta = \_\_\_\_\_ = \_\_\_\_\_$$

✎ Ex. 1 ..... (example2, p676)

Find the angle between $\vec{u} = \vec{i} - 2\vec{j} - 2\vec{k}$ and $\vec{v} = 6\vec{i} + 3\vec{j} + 2\vec{k}$.

sol:

✎ Ex. **2** ..... (example3, p676)

Find the angle θ in the triangle ABC determined by the vertices $A = (0, 0)$, $B = (3, 5)$ and $C = (5, 2)$.

sol:

2 Perpendicular (Orthogonal) Vectors

2.1 *Definition*

Vectors $\vec{u}$ and $\vec{v}$ are _____ (or _____) if and only if _____.

2.2 *Properties of the Dot Product*

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any vectors and c is a scalar, then

1. $\vec{u} \cdot \vec{v} =$ _____
2. $(c\vec{u}) \cdot \vec{v} =$ _____ $=$ _____
3. $\vec{u} \cdot (\vec{v} + \vec{w}) =$ _____
4. $\vec{u} \cdot \vec{u} =$ _____
5. $\vec{0} \cdot \vec{u} =$ _____

2.3 The vector projection of $\vec{u} = \vec{PQ}$ onto a nonzero vector $\vec{v} = \vec{PS}$ is the vector $\vec{PR}$ determined by dropping a perpendicular from Q to the line PS:

_____

2.4 The scalar component of $\vec{u}$ in the direction $\vec{v}$ is the scalar

_____ $=$ _____ $=$ _____

Ex. 3 ..... (example5, p678)

Find the vector projection of $\vec{u} = 6\vec{i} + 3\vec{j} + 2\vec{k}$ onto $\vec{v} = \vec{i} - 2\vec{j} - 2\vec{k}$ and the scalar component of $\vec{u}$ in the direction of $\vec{v}$.

sol:

實習課練習 (EXERCISE 12.3)

□ Find

(a) $\vec{v}\cdot\vec{u}$, $\|\vec{v}\|$, $\|\vec{u}\|$

(b) the cosine of the angle between $\vec{v}$ and $\vec{u}$.

(c) the scalar component of $\vec{u}$ in the direction of $\vec{v}$

(d) the vector $\text{proj}_{\vec{v}}\vec{u}$

4. $\vec{v} = 2\vec{i} + 10\vec{j} - 11\vec{k}$, $\vec{u} = 2\vec{i} + 2\vec{j} + \vec{k}$

8. $\vec{v} =< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} >$, $\vec{u} =< \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}} >$