THOMAS' CALCULUS (12/E)

12.4 The Cross Product

開課班級: (105-2) 通訊1/電機1/智財學程 微積分
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1 The Cross Product of Two Vectors in Space

1.1 *Definition*

Let the unit vector $\vec{n}$ be perpendicular to the plane by the right-hand rule. Then the _____ _____ ($\vec{u}$ cross $\vec{v}$) is the vector defined by

$\vec{u} \times \vec{v} =$ _____

1.2 Nonzero vectors $\vec{u}$ and $\vec{v}$ are _____ if and only if _____.

1.3 *Properties of the Cross Product*

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are any vectors and r, s are scalars, then

1. $(r\vec{u}) \times (s\vec{v}) =$ _____
2. $\vec{u} \times (\vec{v} + \vec{w}) =$ _____
3. $\vec{v} \times \vec{u} =$ _____
4. $(\vec{v} + \vec{w}) \times \vec{u} =$ _____
5. $\vec{0} \times \vec{u} =$ _____
6. $\vec{u} \times (\vec{v} \times \vec{w}) =$ _____

1.4 (a) $\vec{i} \times \vec{j} =$ _____

(b) $\vec{j} \times \vec{k} =$ _____

(c) $\vec{k} \times \vec{i} =$ _____

(d) $\vec{i} \times \vec{i} =$ _____

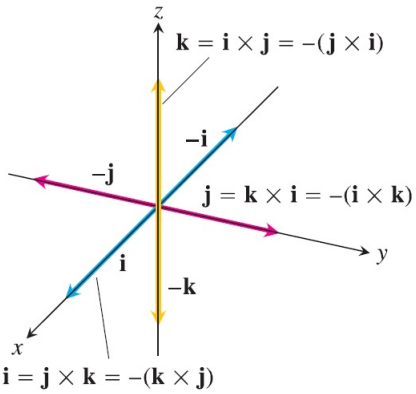

FIGURE 12.29 The pairwise cross products of **i**, **j**, and **k**.

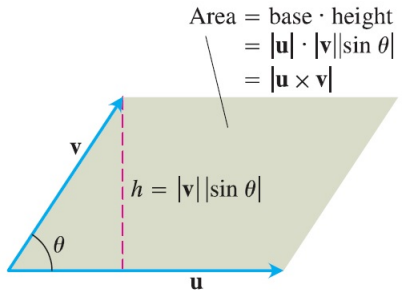

FIGURE 12.30 The parallelogram determined by **u** and **v**.

1.5 If $\vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k}$ and $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$, then

$$\vec{u} \times \vec{v} =$$

pf. _____

1.6 Calculating the triple scalar product as a determinnat.

$$(\vec{u} \times \vec{v}) \cdot \vec{w} =$$

pf. _____

1.7 The magnitude of $\vec{u} \times \vec{v}$ is the area of the _____ determined by $\vec{u}$ and $\vec{v}$, $\vec{u}$ being the _____ of the parallelogram and $\|\vec{v}\|\|\sin\theta\|$ the _____.

$$\|\vec{u} \times \vec{v}\| = \_\_\_\_\_ = \_\_\_\_\_$$

Ex. 1 ..... (example1, p684)

Find $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$ if $\vec{u} = 2\vec{i} + \vec{j} + \vec{k}$ and $\vec{v} = -4\vec{i} + 3\vec{j} + \vec{k}$.

sol:

Ex. 2 ..... (example2, p684)

Find a vector perpendicular to the plane of $P(1, -1, 0)$, $Q(2, 1, -1)$, and $R(-1, 1, 2)$.

sol:

Ex. 3 ..... (example4, p684)

Find a unit vector perpendicular to the plane of $P(1, -1, 0)$, $Q(2, 1, -1)$, and $R(-1, 1, 2)$.

sol:

實習課練習 (EXERCISE 12.4)

☐ In Exercise 1-8, find the length and direction of $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$.

3. $\vec{u} = 2\vec{i} - 2\vec{j} + 4\vec{k}$, $\vec{v} = -\vec{i} + \vec{j} - 2\vec{k}$

6. $\vec{u} = \vec{i} \times \vec{j}$, $\vec{v} = \vec{j} \times \vec{k}$

☐ In Exercise 15-18, (a) Find the area of the triangle determined by the points P, Q and R. (b) Find a unit vector perpendicular to plane PQR.

16. $P(1, 1, 1)$, $Q(2, 1, 3)$, $R(3, -1, 1)$.