統計學 (一)

Anderson's Statistics for Business & Economics (14/E)

Chapter 9: Hypothesis Tests

上課時間地點: 二 D56, 學思 040103
授課教師: 吳漢銘 (國立政治大學統計學系副教授)
教學網站: http://www.hmwu.idv.tw

系級: ____________ 學號: ____________ 姓名: ____________

Overview

1. Statistical inference: how hypothesis testing can be used to determine whether a ________ about the value of a ____________ should or should not be ________.
2. The ______ hypothesis (_____): making a ____________ about a population parameter.
3. The ________ hypothesis (_____): the opposite of what is stated in H_0.
4. The hypothesis testing procedure uses ____________ to test the two competing statements indicated by H_0 and H_a.
5. This chapter shows how hypothesis tests can be conducted about a ____________ and a ____________.

9.1 Developing Null and Alternative Hypotheses

1. It is ____________________ how the null and alternative hypotheses should be formulated.

2. All hypothesis testing applications involve collecting a __________ and using the sample results to provide ____________ for drawing a _____________.

3. In some situations it is easier to identify _______ first and then develop _______.

4. In other situations it is easier to identify _______ first and then develop _______.

The Alternative Hypothesis as a Research Hypothesis

1. Many applications of hypothesis testing involve an attempt to gather evidence in ________________________________. In these situations, it is often best to begin with the _____________ hypothesis and make it the conclusion that the researcher ___________________.

2. Example Consider a particular automobile that currently attains a fuel efficiency of 24 miles per gallon in city driving.

   (a) *Goal*: A product research group has developed a ________ fuel injection system (燃料噴射系統) designed to ___________ the miles-per-gallon rating. The group will run controlled tests with the new fuel injection system looking for statistical support for the conclusion that the new fuel injection system provides more miles per gallon than the current system.

   (b) Several new fuel injection units will be manufactured, installed in test automobiles, and subjected to research-controlled driving conditions.

   (c) The ______________________________ for these automobiles will be computed and used in a hypothesis test to determine if it can be concluded that the new system provides ______________________________.

   (d) In terms of the population mean miles per gallon _____, the research hypothesis __________ becomes the alternative hypothesis.

(e) Since the current system provides an average or mean of 24 miles per gallon, we will make the tentative assumption that the new system is not any better than the current system and choose __________ as the null hypothesis.

____

(f) If the sample results lead to the conclusion to reject H_0, the inference can be made that _______________ is true.

(g) The researchers have the ____________________ to state that the new fuel injection system increases the mean number of miles per gallon.

(h) If the sample results lead to the conclusion that H_0 cannot be rejected, the researchers cannot conclude that the new fuel injection system is better than the current system. Production of automobiles with the new fuel injection system on the basis of better gas mileage cannot be justified. Perhaps ____________________ ____________________ can be conducted.

3. Before adopting something ________ (e.g., products, methods, systems), it is desirable to conduct research to determine if there is ____________________ for the conclusion that the new approach is indeed better. In such cases, the research hypothesis is stated as the ________________________.

   (a) Example A new teaching method is developed that is believed to be better than the current method.

      i. H_0: the new method is no better than the old method.

      ii. H_a: the new method is __________.

   (b) Example A new sales force bonus plan is developed in an attempt to increase sales.

      i. H_0: the new bonus plan does not increase sales.

      ii. H_a: the new bonus plan _________________.

   (c) Example A new drug is developed with the goal of lowering blood pressure more than an existing drug.

      i. H_0: the new drug does not provide lower blood pressure than the existing drug.

ii. H_a: the new drug __________ blood pressure ______________ the existing drug.

4. In each case, ____________ of the null hypothesis H_0 provides ____________________ for the research hypothesis.

The Null Hypothesis as an Assumption to Be Challenged

1. The situations below that it is helpful to develop the null hypothesis first.

   (a) Consider applications of hypothesis testing where we begin with a __________ or an _______________ that a statement about the value of a ______________ ______________ is ________.

   (b) We will then use a hypothesis test to ___________________________ and determine if there is statistical evidence to conclude that the assumption is ____________.

2. The null hypothesis H_0 expresses the ______________________ about the value of the population parameter. The alternative hypothesis H_a is that the belief or assumption is ____________.

3. Example Consider the situation of a manufacturer of soft drink products.

   (a) The label on a soft drink bottle states that it contains 67.6 fluid ounces. We consider the label correct provided the ___________________ filling weight for the bottles is ___________ 67.6 fluid ounces.

   (b) We would begin with the _______________ that the label is correct and state the null hypothesis as ____________.

   (c) The challenge to this assumption would imply that the label is incorrect and the bottles are being under-filled. This challenge would be stated as the alternative hypothesis ____________.

   ________________________________.

   (d) A government agency with the responsibility for validating manufacturing labels could select a sample of soft drinks bottles, compute the ________________ filling weight, and use the sample results to test the preceding hypotheses.

(e) If the sample results lead to the conclusion to ____________, the inference that ______________________ can be made. With this statistical support, the agency is justified in concluding that the __________________ and ______________ of the bottles is occurring.

(f) If the sample results indicate _______________________, the assumption that the manufacturer's labeling is correct cannot be rejected. With this conclusion, ____________ would be taken.

4. $\boxed{\text{Example}}$ Consider the soft drink bottle filling example from the manufacturer′ s point of view.

(a) The bottle-filling operation has been designed to fill soft drink bottles with 67.6 fluid ounces as stated on the label.

i. The company does not want to ___________ the containers because that could result in an underfilling complaint from customers or, perhaps, a government agency.

ii. However, the company does not want to __________ containers either because putting more soft drink than necessary into the containers would be an unnecessary cost.

(b) The company's goal would be to adjust the bottle-filling operation so that the population mean filling weight per bottle is 67.6 fluid ounces as specified on the label.

(c) In a hypothesis testing application, we would begin with the assumption that the production process is operating correctly and state the null hypothesis as ___________ fluid ounces.

(d) The alternative hypothesis that challenges this assumption is that ____________, which indicates either overfilling or underfilling is occurring.

____

(e) Suppose that the soft drink manufacturer uses a quality control procedure to periodically select a sample of bottles from the filling operation and computes the _______________ filling weight per bottle.

i. If the sample results lead to the conclusion to __________, the inference is made that ______________ is true. We conclude that the bottles are not being filled properly and the ________________________________ to restore the population mean to 67.6 fluid ounces per bottle.

ii. If the sample results indicate ____________________, the assumption that the manufacturer's bottle filling operation is functioning properly cannot be rejected. In this case, ________________ would be taken and the production operation would continue to run.

5. The two preceding forms of the soft drink manufacturing hypothesis test show that the null and alternative hypotheses may ________________________________ of the researcher or decision maker.

6. To correctly ____________________ it is important to understand the context of the situation and structure the hypotheses to provide the information the researcher or decision maker wants.

Summary Of Forms for Null and Alternative Hypotheses

1. Depending on the situation, hypothesis tests about a population parameter (the population mean and the population proportion) may take one of three forms:

   ____

2. The first two forms are called ________________. The third form is called a ________________.

3. The __________ part of the expression (either $\geq$, $\leq$, or $=$) always appears in the _______ hypothesis.

4. In selecting the proper form of H_0 and H_a, keep in mind that the ____________ hypothesis is often what ______________________________. Hence, asking whether the user is looking for evidence to support ________________________ will help determine H_a.

☺ **EXERCISES 9.1**: 2, 3

9.2 Type I and Type II Errors

1. Ideally the hypothesis testing procedure should lead to the ________________ when ____________ and the rejection of H_0 when H_a is true.

2. (Table 9.1) The correct conclusions are not always possible.

TABLE 9.1 Errors and Correct Conclusions in Hypothesis Testing

		Population Condition	
		H_0 True	H_a True
Conclusion	Accept H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

   (a) We reject H_0 if H_0 is true, we make a ______________.

   (b) If H_a is true, the conclusion is correct when we reject H_0.

   (c) If H_0 is true, the conclusion is correct when we accept H_0.

   (d) If H_0 is false (H_a is true), we make a ______________ when we accept H_0.

3. Example An automobile product research group developed a new fuel injection system designed to increase the miles-per-gallon rating of a particular automobile.

   (a) With the current model obtaining an average of 24 miles per gallon, the hypothesis test was formulated as follows.

   ____

   (b) The alternative hypothesis, $H_a : \mu > 24$, indicates that the researchers are looking for __________________________ the conclusion that the population mean miles per gallon with the new fuel injection system is ______________ 24.

   (c) *Type I error*: __________________________ corresponds to the researchers ___________ that the new system improves the miles-per-gallon rating ($\mu > 24$) when _________ the new system is not any better than the current system.

(d) *Type II error*: ____________________ corresponds to the researchers __________ that the new system is not any better than the current system ($\mu \leq 24$) when ________ the new system improves miles-per-gallon performance.

4. **Level of Significance**: The level of significance is the probability of making a Type I error when the null hypothesis is true as an __________.

   (a) $\boxed{\text{Example}}$ For the miles-per-gallon rating hypothesis test, the null hypothesis is $H_0 : \mu \leq 24$. Suppose the null hypothesis is true as an __________; that is, ________. The level of significance is the probability of ____________________ when ________.

   (b) The Greek symbol _____ (alpha) is used to denote the level of significance, and common choices for α are ______________.

   (c) In practice, the person responsible for the hypothesis test specifies the level of significance. By selecting α, that person is ____________________ of making a Type I error.

   (d) If the cost of making a Type I error is ________________, ______________ values of α are preferred.

5. **The significance tests**: Applications of hypothesis testing that only control for the Type I error are called ________________.

6. Although most applications of hypothesis testing control for the probability of making a Type I error, they do not always control for the probability of making a ______________.

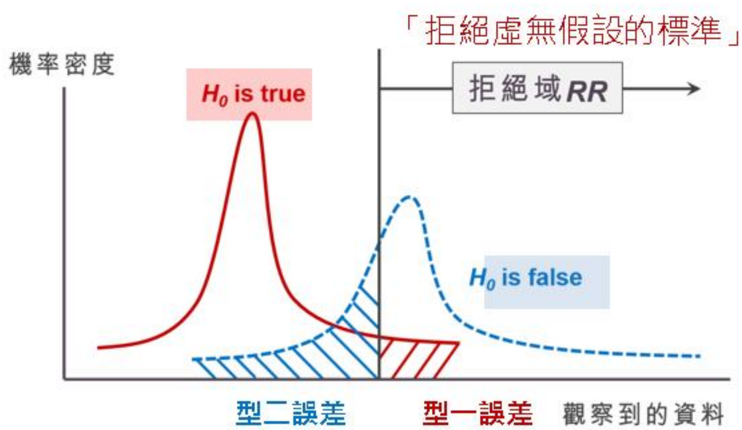
   (a) Hence, if we decide to accept H_0, we cannot determine ______________ we can be with that decision. Because of the ____________ associated with making a Type II error when conducting significance tests, statisticians usually recommend that we use the statement ________________ instead of ____________

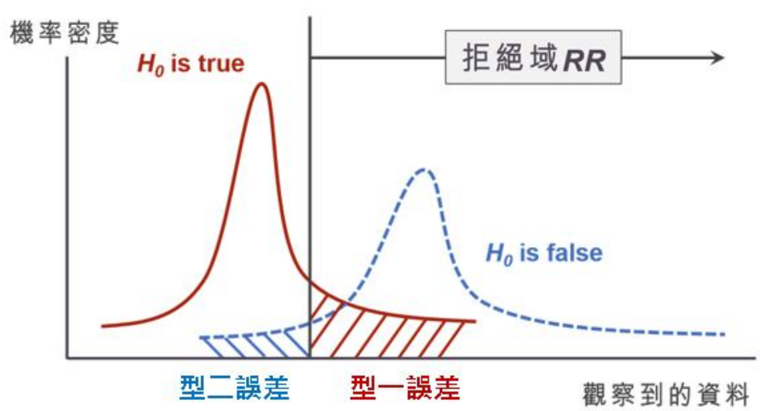
   (b) Using the statement "do not reject H_0" carries the recommendation to __________ ______________________. In effect, by not directly accepting H_0, the statistician ______________ of making a Type II error.

(c) Whenever the probability of making a Type II error has not been determined and controlled, we will not make the statement ____________ In such cases, only two conclusions are possible: ______________ or __________.

(d) Although controlling for a Type II error in hypothesis testing is ____________, it can be done. In Sections 9.7 and 9.8 we will illustrate procedures for determining and controlling the probability of making a Type II error. If proper controls have been established for this error, ________ based on the ____________ conclusion can be appropriate.

☺ **EXERCISES 9.2**: 6, 7





9.3 Population Mean: σ Known

One-tailed Test

1. One-tailed tests about a population mean take one of the following two forms:

____

2. Example The Federal Trade Commission (FTC) periodically conducts statistical studies designed to test the claims that manufacturers make about their products. For example, the label on a large can of Hilltop Coffee states that the can contains 3 pounds of coffee. The FTC knows that Hilltop's production process cannot place exactly 3 pounds of coffee in each can, even if the mean filling weight for the population of all cans filled is 3 pounds per can ($\mu_0 = 3$). However, as long as the population mean filling weight is at least 3 pounds per can, the rights of consumers will be protected. Thus, the FTC interprets the label information on a large can of coffee as a claim by Hilltop that the population mean filling weight is at least 3 pounds per can. We will show how the FTC can check Hilltop's claim by conducting a ______________________.

   (a) *Develop the null and alternative hypotheses for the test.* If the population mean filling weight is at least 3 pounds per can, Hilltop's claim is correct:

   $$H_0 : \_\_\_\_\_\_\_\_ \qquad H_a : \_\_\_\_\_\_\_\_$$

      i. If the sample data indicate that ____________________, no action should be taken against Hilltop.

      ii. If the sample data indicate __________________, $H_a : \mu < 3$, is true. A conclusion of ____________ and a charge of a label violation against Hilltop would be justified.

      iii. Suppose a sample of $n = 36$ cans of coffee is selected and the sample mean _____ is computed as an estimate of the population mean _____. If the value of the sample mean $\bar{x}$ is __________ 3 pounds, the sample results will ____________ on the null hypothesis.

(b) *Specifying the level of significance, α:*

i. (Recall) The level of significance is the probability of making a Type I error by ______________ when H_0 is true as an ___________.

ii. If the ________ of making a Type I error is _________________, a _________ ________________ should be chosen for the level of significance.

iii. In the Hilltop Coffee study, the director of the FTC's testing program made the following statement: "If the company is meeting its weight specifications at $\mu = 3$, I do not want to take action against them. But, I am willing to risk a 1% chance of making such an error." From the director's statement, we set the level of significance for the hypothesis test at ____________.

iv. Thus, we must design the hypothesis test so that the probability of making a ______________ when $\mu = 3$ is 0.01.

3. By developing the null and alternative _____________ and specifying the __________ ______________ for the test, we carry out the first two steps required in conducting every hypothesis test. We are now ready to perform the third step of hypothesis testing: ___________________ data and compute the value of what is called a _______________.

Test statistic

1. $\boxed{\text{Example}}$ For the Hilltop Coffee study, previous FTC tests show that the population standard deviation can be assumed __________ with a value of $\sigma = 0.18$. These tests also show that the population of filling weights can be assumed to have a __________ distribution.

2. The sampling distribution of $\bar{x}$ is ____________ distributed with a known value of $\sigma = 0.18$ and a sample size of $n = 36$.

3. (Figure 9.1) the sampling distribution of $\bar{x}$ when the null hypothesis is true as an equality ($\mu = \mu_0 = 3$). The standard error of $\bar{x}$ is given by $\sigma_{\bar{x}} =$ __________________________. The sampling distribution of

$$z = \underline{\hspace{5cm}}$$ is a standard normal distribution.

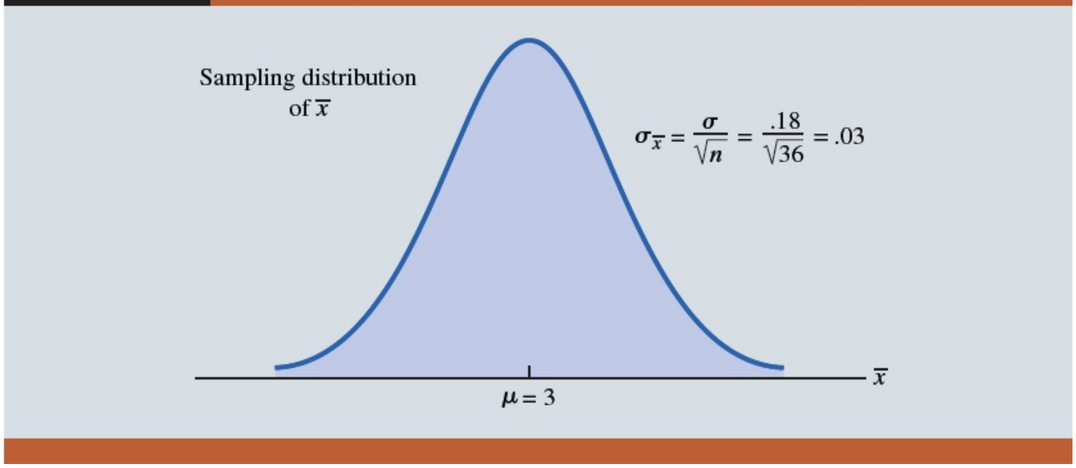


FIGURE 9.1 Sampling Distribution of $\bar{x}$ for the Hilltop Coffee Study When the Null Hypothesis Is True as an Equality ($\mu = 3$)

4. A value of ________________ means that the value of $\bar{x}$ is ____________________ ________ below the hypothesized value of the mean.

5. The lower tail area at $z = -3.00$ is _________. Hence, the probability of obtaining a value of z that is three or more standard errors ________________ is 0.0013.

6. The probability of obtaining a value of $\bar{x}$ that is 3 or more standard errors below the hypothesized population mean $\mu_0 = 3$ is also 0.0013. Such a result is __________ if the null hypothesis is true.

7. For hypothesis tests about a population mean in the σ known case, we use the standard normal random variable _____ as a ______________ to determine whether $\bar{x}$ _______________ the hypothesized value of μ enough to justify rejecting the null hypothesis.

8. **Test Statistic for Hypothesis Tests About a Population Mean: σ Known**

$$\frac{\qquad\qquad}{\qquad\qquad\qquad} .$$

9. The key question for a lower tail test is, ____________ must the test statistic z be before we choose to ___________? Two approaches: the __________ approach and the ______________ approach.

p-value approach

1. **p-value**: A p-value is a probability that provides a ________ of the evidence ________ provided by the ________.
   (a) The p-value is used to determine whether H_0 should be ________.
   (b) A ________ indicates the value of the test statistic is ________ given the assumption that ________.
   (c) ________ p-values indicate ________ against H_0.
2. The value of the test statistic is used to compute the p-value.
   (a) For a ________ test, the p-value is the probability of obtaining a value for the test statistic ________ or ________ than that provided by the sample.
   (b) To compute the p-value for the lower tail test in the σ known case, we use the standard normal distribution to find the probability that ____ is ________ the value of the test statistic.
   (c) After computing the p-value, we must then decide whether it is ________ to reject the null hypothesis; as we will show, this decision involves comparing the p-value to the level of significance.

Question .... (p427)

Suppose the sample of 36 Hilltop coffee cans provides a sample mean of $\bar{x} = 2.92$ pounds. Is $\bar{x} = 2.92$ small enough to cause us to reject H_0? Compute the p-value for the Hilltop Coffee lower tail test.

sol:

– Because this is a lower tail test, the p-value is the area under the standard normal curve for values of $z \leq$ the value of the test statistic.

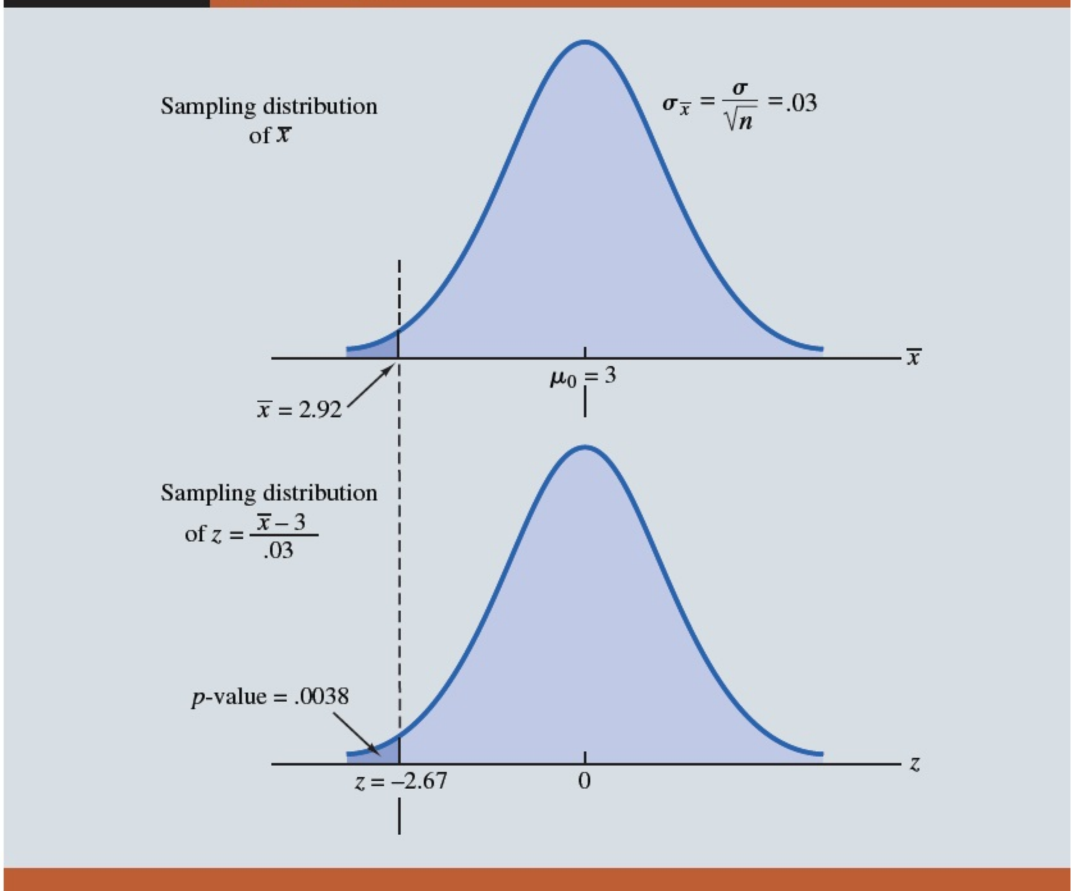
- Using $\bar{x} = 2.92$, $\sigma = 0.18$, and $n = 36$, the value of the test statistic z:

$$z = \frac{\qquad\qquad}{\qquad\qquad}.$$

- p-value ____________________.

- (Figure 9.2) $\bar{x} = 2.92$ corresponds to $z = -2.67$ and a p-value $= 0.0038$.

FIGURE 9.2 p-Value for the Hilltop Coffee Study When $\bar{x} = 2.92$ and $z = -2.67$



3. This p-value (0.0038) indicates a small probability of ____________________ of $\bar{x} = 2.92$ (and a test statistic of -2.67) or smaller when sampling from a population with ________.

4. This p-value does not provide much support for the null hypothesis, but ______________ enough to cause us to reject H_0? The answer depends upon ______ for the test.

5. As noted previously, the director of the FTC's testing program selected a value of 0.01 for the level of significance means that the director is ____________________ a probability of 0.01 of rejecting H_0 when it is true as an ____________________.

6. The sample of 36 coffee cans in the Hilltop Coffee study resulted in a p-value = 0.0038, which means that the ______________ of obtaining a value of $\bar{x} = 2.92$ or less when H_0 is true as an equality is __________.

7. Because ______________________________, we _____________. Therefore, we find ______________________________ to reject the null hypothesis at the 0.01 level of significance.

8. **Rejection Rule Using p-value**. For a level of significance α, the rejection rule using the p-value approach is:

   ____

9. $\boxed{\text{Example}}$ In the Hilltop Coffee test, the p-value of 0.0038 resulted in the rejection of H_0. The observed p-value of 0.0038 means that we would reject H_0 for any value of ______________. For this reason, the p-value is also called ______________________ _______________.

10. Different decision makers may express different opinions concerning the ___________ _________________________ and may choose a ______________.

Critical value approach

1. The critical value is the value of the test statistic that corresponds to an _____________ in the lower tail of the ________________________ of the test statistic.

2. The critical value is the ________________ of the test statistic that will result in the rejection of the null hypothesis.

3. For a lower tail test, the critical value serves as a ______________ for determining whether the value of the test statistic is ________________ to reject the null hypothesis.

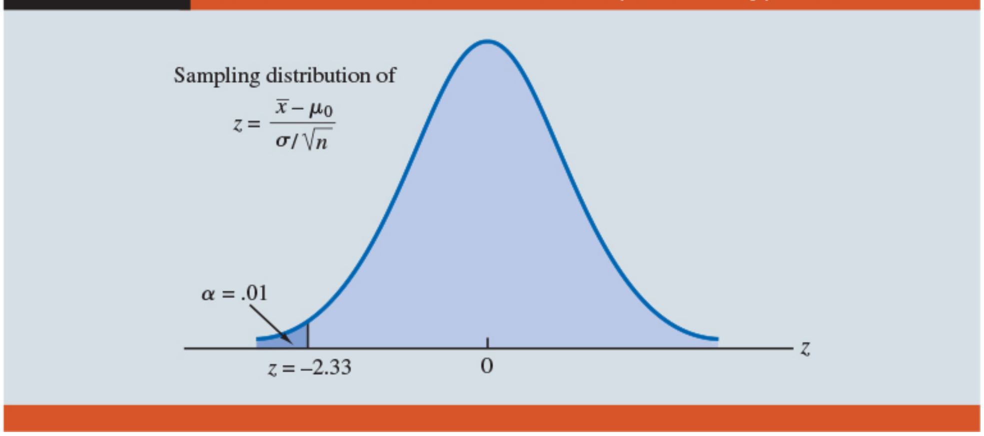
4. $\boxed{\text{Example}}$ the Hilltop Coffee example.

   (a) In the σ known case, the sampling distribution for the test statistic z is a ________________ distribution. Therefore, the critical value is the value of the test statistic that corresponds to an area of __________ in the lower tail of a standard normal distribution.

   (b) (Figure 9.3) We find that _______________ provides an area of 0.01 in the lower tail, ____________________.

   (c) If the sample results in a value of the test statistic that is less than or equal to -2.33, the corresponding p-value will be less than or equal to 0.01; in this case, we should reject H_0.

FIGURE 9.3 Critical Value = −2.33 for the Hilltop Coffee Hypothesis Test



   (d) Hence, for the Hilltop Coffee study the critical value ______________ for a level of significance of 0.01 is

   Reject H_0 if ____________

   (e) In the Hilltop Coffee example, $\bar{x} = 2.92$ and the test statistic is $z = -2.67$. Because __________________, we can reject H_0 and conclude that Hilltop Coffee is _____________ cans.

5. **Rejection Rule for a Lower Tail Test: Critical Value Approach**. We can generalize the rejection rule for the critical value approach to handle any level of significance. The rejection rule for a lower tail test follows.

   Reject H_0 if __________

where $-z_\alpha$ is the ________________; that is, the z value that provides an area of α in the lower tail of the standard normal distribution.

Summary

1. The p-value approach to hypothesis testing and the critical value approach will always lead to ____________ rejection decision.
2. The advantage of the p-value approach is that the p-value tells us __________________ the results are (the observed level of significance).
3. If we use the critical value approach, we only know that the results are significant __________________.
4. We can use the same general approach to conduct an _________________. The test statistic z is still computed using equation (9.1). But, for an upper tail test, the p-value is the probability of obtaining a value for the test statistic ________________ ______________ that provided by the sample.
5. To compute the p-value for the upper tail test in the σ known case, we must use the standard normal distribution to find the probability that z is __________________________ the value of the test statistic.
6. **Computation of p-Values for One-Tailed Tests**
   (a) Compute the value of the test statistic using equation (9.1):

   $$z = \frac{\qquad}{\qquad}$$

   (b) *Lower tail test*: Using the standard normal distribution, compute the probability that z is ______________________ the value of the test statistic (area in the _________ tail).

   (c) *Upper tail test*: Using the standard normal distribution, compute the probability that z is _________________________ the value of the test statistic (area in the _________ tail).

Two-tailed Test

1. The general form for a two-tailed test about a population mean:

   ____

2. $\boxed{\text{Example}}$ The U.S. Golf Association (USGA) establishes rules that manufacturers of golf equipment must meet if their products are to be acceptable for use in USGA events. MaxFlight Inc. uses a high-technology manufacturing process to produce golf balls with a mean driving distance of 295 yards. Sometimes, however, the process gets out of adjustment and produces golf balls with a mean driving distance different from 295 yards. When the mean distance falls below 295 yards, the company worries about losing sales because the golf balls do not provide as much distance as advertised. When the mean distance passes 295 yards, MaxFlight's golf balls may be rejected by the USGA for exceeding the overall distance standard concerning carry and roll. MaxFlight's quality control program involves taking periodic samples of 50 golf balls to monitor the manufacturing process. For each sample, a hypothesis test is conducted to determine whether the process has fallen out of adjustment.

   (a) We begin by assuming that the process is functioning correctly; that is, the golf balls being produced have a mean distance of 295 yards. This assumption establishes the null hypothesis. The alternative hypothesis is that the mean distance is not equal to 295 yards.

   ______________________________.

   (b) If the sample mean $\bar{x}$ is ____________________ than 295 yards or ________________________ than 295 yards, we will reject H_0. In this case, corrective action will be taken to adjust the manufacturing process.

   (c) If $\bar{x}$ does not deviate from the hypothesized mean $\mu_0 = 295$ by a significant amount, H_0 will not be rejected and ______________ will be taken to adjust the manufacturing process.

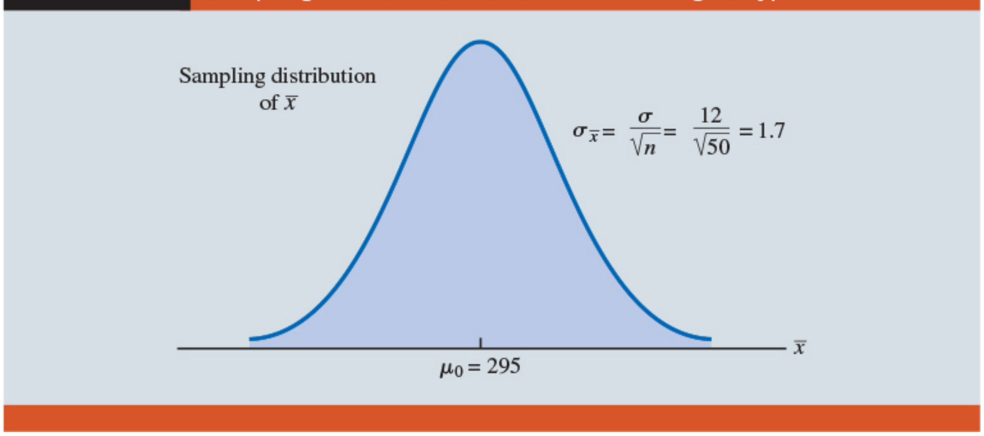
   (d) The quality control team selected $\alpha = 0.05$ as the level of significance for the test. Data from previous tests conducted when the process was known to be in adjustment show that the population standard deviation can be assumed known with a value of __________. Thus, with a sample size of $n = 50$, the standard error of $\bar{x}$ is

$$\sigma_{\bar{x}} = \underline{\hspace{4cm}}$$

(e) Because the sample size is large, the ____________________ allows us to conclude that the sampling distribution of $\bar{x}$ can be approximated by a ____________________.

(f) (Figure 9.4) the sampling distribution of $\bar{x}$ for the MaxFlight hypothesis test with a hypothesized population mean of $\mu_0 = 295$.

FIGURE 9.4 Sampling Distribution of $\bar{x}$ for the MaxFlight Hypothesis Test



3. Suppose that a sample of 50 golf balls is selected and that the sample mean is $\bar{x} = 297.6$ yards. This sample mean provides support for the conclusion that the population mean is larger than 295 yards. Is this value of $\bar{x}$ ____________________ 295 to cause us to reject H_0 at the 0.05 level of significance?

p-value approach

1. (Recall) the p-value is a probability used to determine whether the null hypothesis should be rejected.

2. For a two-tailed test, values of the test statistic ________________ provide evidence against the null hypothesis.

3. For a two-tailed test, the p-value is the probability of obtaining a value for the test statistic ________ or ________ that provided by the sample.

4. $\boxed{\text{Example}}$ the MaxFlight hypothesis test example.

   (a) *Compute the value of the test statistic.* For the σ known case, the test statistic z is a standard normal random variable. Using equation (9.1) with $\bar{x} = 297.6$, the value of the test statistic is

   $$z = \frac{}{}.$$

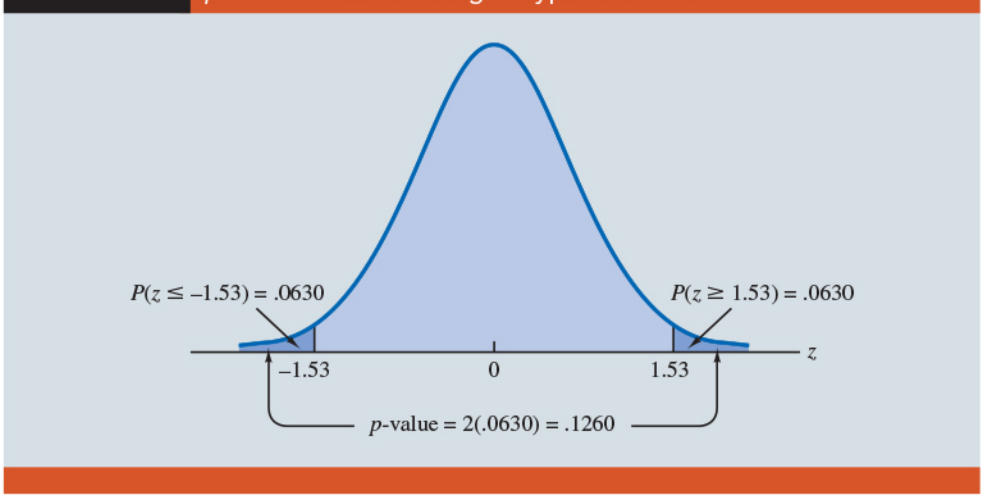
   (b) *Compute the p-value.* Find the probability of obtaining a value for the test statistic ________. Clearly values of ________ are at least as unlikely.

   (c) But, because this is a ________ test, values of ________ are also at least as unlikely as the value of the test statistic provided by the sample.

   (d) (Figure 9.5) the two-tailed p-value: ________.

   (e) Because the normal curve is symmetric, ________. Thus, the upper tail area is $P(z \geq 1.53) =$ ________.

FIGURE 9.5 *p*-Value for the MaxFlight Hypothesis Test



   (f) The p-value for the MaxFlight two-tailed hypothesis test is

   $$p\text{-value} = \underline{}.$$

(g) With a level of significance of $\alpha = 0.05$, we ______________ because the ______________. Because the null hypothesis is not rejected, no action will be taken to adjust the MaxFlight manufacturing process.

5. **Computation of p-Values for Two-Tailed Tests**.

   (a) Compute the value of the test statistic using equation (9.1).

   (b) If the value of the test statistic is in the ______________, compute the probability that z is ______________ the value of the test statistic (the upper tail area).

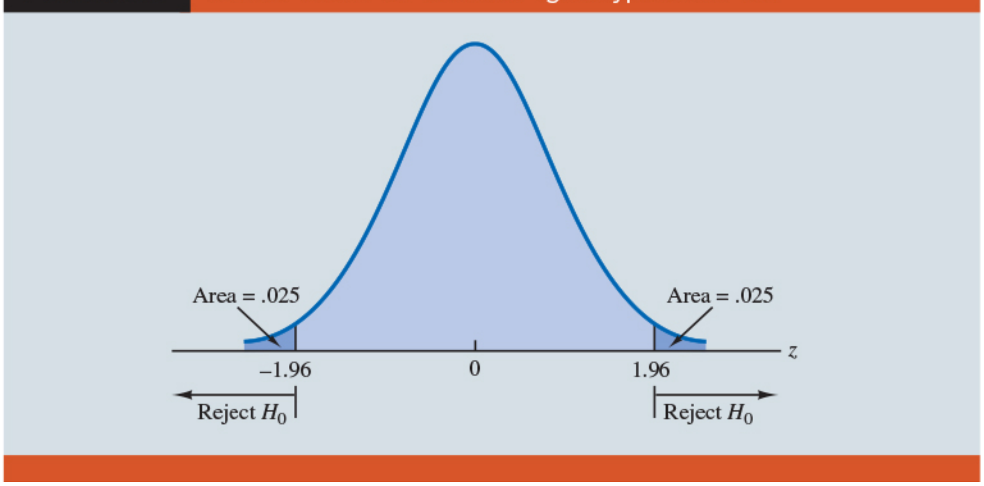
   (c) If the value of the test statistic is in the ______________, compute the probability that z is ______________ the value of the test statistic (the lower tail area).

   (d) Double the probability (or tail area) from step (b) or (c) to obtain the p-value.

Critical value approach

1. (Figure 9.6) the critical values for the test will occur in both the lower and upper tails of the standard normal distribution. With a level of significance of $\alpha = 0.05$, the area in each tail corresponding to the critical values is ______________.

FIGURE 9.6 Critical Values for the MaxFlight Hypothesis Test



2. The critical values for the test statistic are ______________ and ______________.

3. The two-tailed rejection rule is

$$\underline{\hspace{6cm}}$$

4. Because the value of the test statistic for the MaxFlight study is $z = 1.53$, the statistical evidence will not permit us to reject the null hypothesis at the 0.05 level of significance.

Summary and Practical Advice

1. Summary of the hypothesis testing procedures about a population mean for the σ known case. Note that μ_0 is the hypothesized value of the population mean.

TABLE 9.2 Summary of Hypothesis Tests About a Population Mean: σ Known Case

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	H_0: $\mu \geq \mu_0$ H_a: $\mu < \mu_0$	H_0: $\mu \leq \mu_0$ H_a: $\mu > \mu_0$	H_0: $\mu = \mu_0$ H_a: $\mu \neq \mu_0$
Test Statistic	$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $z \leq -z_\alpha$	Reject H_0 if $z \geq z_\alpha$	Reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$

2. **Steps of Hypothesis Testing**

Step 1. Develop the null and alternative hypotheses (__________).

Step 2. Specify the level of significance (______).

Step 3. Collect the sample data (____________________) and compute the value of the test statistic (____________).

***p*-value Approach**

Step 4. Use the value of the test statistic to compute the p-value.

Step 5. Reject H_0 if the ______________.

Step 6. Interpret the ______________________ in the context of the application.

Critical Value Approach

Step 4. Use α to determine the critical value (______ or _______) and the rejection rule.

Step 5. Use the value of the test statistic and the rejection rule to determine whether to reject H_0.

Step 6. Interpret the statistical conclusion in the context of the application.

3. Practical advice about the sample size for hypothesis tests is similar to the advice we provided about the sample size for interval estimation in Chapter 8.

   (a) In most applications, a sample size of __________ is adequate when using the hypothesis testing procedure described in this section.

   (b) If the population is ____________ distributed, the hypothesis testing procedure that we described is _________ and can be used for any sample size.

   (c) If the population is not normally distributed but is at least ____________________, sample sizes ________________ can be expected to provide acceptable results.

Relationship Between Interval Estimation and Hypothesis Testing

1. (Recall, Chapter 8) For the σ known case, the $(1-\alpha)\%$ confidence interval estimate of a population mean is given by

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

2. (Recall, Chapter 9) a two-tailed hypothesis test about a population mean:

$$H_0 : \mu = \mu_0, \quad H_a : \mu \neq \mu_0$$

   where μ_0 is the hypothesized value for the population mean.

3. Constructing a $100(1-\alpha)\%$ confidence interval for the population mean: ____________ of the confidence intervals generated ____________ the population mean and ________ of the confidence intervals generated ______________ the population mean.

4. If we reject H_0 whenever the confidence interval does not contain μ_0, we will be rejecting H_0 when it is true ($\mu = \mu_0$) with probability α.

5. Recall that α is the probability of rejecting the null hypothesis when it is true.

6. So constructing a ____________ confidence interval and rejecting H_0 whenever the interval does not contain μ_0 is ___________ to conducting a ___________ hypothesis test with _____ as the level of significance.

7. A Confidence Interval Approach to Testing a Hypothesis of the Form:

$$H_0 : \mu = \mu_0, \quad H_a : \mu \neq \mu_0$$

   (a) Select a simple random sample from the population and use the value of the sample mean $\bar{x}$ to develop the confidence interval for the population mean μ.

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

   (b) If the confidence interval contains the hypothesized value______, do not reject H_0. Otherwise, reject H_0

8. Note that this discussion and example pertain to two-tailed hypothesis tests about a population mean. However, the same confidence interval and two-tailed hypothesis testing relationship exists for other population parameters.

9. The relationship can also be extended to one-tailed tests about population parameters. Doing so, however, requires the development of _________________________, which are rarely used in practice.

Question .... (p435)

The MaxFlight hypothesis test takes the following form:

$$H_0 : \mu = 295, \quad H_a : \mu \neq 295.$$

Conducting the MaxFlight hypothesis test with a level of significance of $\alpha = 0.05$ using the confidence interval approach.

sol:

- We sampled ___________ golf balls and found a sample mean distance of _____________ yards. Recall that the population standard deviation is ___________ .

- The 95% confidence interval estimate of the population mean is

  ____

$$\begin{aligned} 297.6 &\pm 1.96\frac{12}{\sqrt{50}} \\ 297.6 &\pm 3.3 \quad \mathit{or} \quad (294.3, 300.9). \end{aligned}$$

- With 95% confidence, the mean distance for the population of golf balls is between 294.3 and 300.9 yards.
- Because the interval contains the hypothesized value for the population mean, $\mu_0 = 295$, the hypothesis testing conclusion is that the null hypothesis, $H_0 : \mu = 295$, cannot be rejected.

☺ **EXERCISES 9.3**: 9, 11, 15, 18, 22

9.4 Population Mean: σ Unknown

1. To conduct a hypothesis test about a population mean for the σ unknown case, the sample mean ______ is used as an estimate of ______ and the sample standard deviation ______ is used as an estimate of ______.

2. (Recall) For the σ known case, the sampling distribution of the test statistic has a ____________________ distribution. For the σ unknown case, however, the sampling distribution of the test statistic follows the ________________; it has slightly more variability because the sample is used to develop estimates of both μ and σ.

3. **Test Statistic for Hypothesis Tests about a Population Mean: σ Unknown** the test statistic has a t distribution with $n-1$ degrees of freedom:

   (9.2)

4. The t distribution is based on an assumption that the ______________ from which we are sampling has a ___________ distribution. However, research shows that this assumption can be relaxed considerably when the sample size is ________________.

One-tailed Test

1. $\boxed{\text{Example}}$ A business travel magazine wants to classify transatlantic gateway airports according to the mean rating for the population of business travelers. A rating scale with a low score of 0 and a high score of 10 will be used, and airports with a population mean rating greater than 7 will be designated as superior service airports. The magazine staff surveyed a sample of 60 business travelers at each airport to obtain the ratings data. The sample for London's Heathrow Airport provided a sample mean rating of $\bar{x} = 7.25$ and a sample standard deviation of $s = 1.052$. Do the data indicate that Heathrow should be designated as a superior service airport?

   (a) We want to develop a hypothesis test for which the decision to reject H_0 will lead to the conclusion that the population mean rating for the Heathrow Airport is greater than 7.

   (b) The null and alternative hypotheses for this upper tail test:

(c) Use $\alpha = 0.05$, with $\bar{x} = 7.25$, $\mu_0 = 7$, $s = 1.052$, and $n = 60$, the value of the test statistic:

$$t = \underline{\hspace{6cm}}$$

(d) The sampling distribution of t has ______________ degrees of freedom. Because the test is an upper tail test, the p-value is __________, that is, the upper tail area corresponding to the value of the test statistic.

(e) (Table 2 in Appendix B) the t distribution with 59 degrees of freedom provides the following information.

Area in Upper Tail	0.20	0.10	0.05	0.025	0.01	0.005
t-Value (59 df)	0.848	1.296	1.671	2.001	2.391	2.662

i. We see that $t = 1.84$ is between ____________. The values in the "Area in Upper Tail" row show that the p-value must be less than ______ and greater than ______.

ii. With a level of significance of $\alpha = 0.05$, this placement is all we need to know to make the decision to reject the null hypothesis and conclude that Heathrow should be classified as a superior service airport.

(f) (Using software) $t = 1.84$ provides the upper tail p-value of _______ for the Heathrow Airport hypothesis test. With ___________, we reject the null hypothesis and conclude that Heathrow should be classified as a superior service airport.

(g) The critical value corresponding to an area of $\alpha = 0.05$ in the upper tail of a t distribution with 59 degrees of freedom is ____________.

(h) The rejection rule using the critical value approach is to reject H_0 if $t \geq 1.671$. Because _____________, H_0 is rejected. Heathrow should be classified as a superior service airport.

Two-tailed Test

1. $\boxed{\text{Example}}$ Consider the hypothesis testing situation facing Holiday Toys. The company manufactures and distributes its products through more than 1000 retail outlets. In planning production levels for the coming winter season, Holiday must decide how many units of each product to produce prior to knowing the actual

demand at the retail level. For this year's most important new toy, Holiday′ s marketing director is expecting demand to average 40 units per retail outlet. Prior to making the final production decision based upon this estimate, Holiday decided to survey a sample of 25 retailers in order to develop more information about the demand for the new product. Each retailer was provided with information about the features of the new toy along with the cost and the suggested selling price. Then each retailer was asked to specify an anticipated order quantity. With μ denoting the population mean order quantity per retail outlet, the sample data will be used to conduct the following two-tailed hypothesis test:

____

(a) If H_0 cannot be rejected, Holiday will continue its production planning based on the marketing director's estimate that the population mean order quantity per retail outlet will be $\mu = 40$ units.

(b) If H_0 is rejected, Holiday will immediately reevaluate its production plan for the product.

(c) A two-tailed hypothesis test is used because Holiday wants to reevaluate the production plan if the population mean quantity per retail outlet is less than anticipated or greater than anticipated.

(d) Because no historical data are available (it's a new product), the population mean μ and the population standard deviation must both be estimated using ____________ from the sample data.

(e) The sample of 25 retailers provided a mean of $\bar{x} = 37.4$ and a standard deviation of $s = 11.79$ units.

(f) (*Check on the form of the population distribution*). The histogram of the sample data showed no evidence of skewness or any extreme outliers, so the analyst concluded that the use of the ________________ with $n-1 = 24$ degrees of freedom was appropriate.

(g) Using equation (9.2) with $\bar{x} = 37.4$, $\mu_0 = 40$, $s = 11.79$, and $n = 25$, the value of the test statistic is

$$t = \underline{\hspace{8cm}}\,.$$

(h) The t distribution table only contains positive t values. Because the t distribution is ____________, however, the upper tail area at __________ is the same as the lower tail area at ____________.

(i) (Table 2 in Appendix B)

Area in Upper Tail	0.20	0.10	0.05	0.025	0.01	0.005
t-Value (24 *df*)	0.857	1.318	1.711	2.064	2.492	2.797

(j) We see that $t = 1.10$ is between ________________. From the "Area in Upper Tail" row, we see that the area in the upper tail at $t = 1.10$ is between ______________.

(k) When we double these amounts, we see that the p-value must be between _______ and _______. With a level of significance of $\alpha = 0.05$, we now know that the p-value is greater than α. Therefore, H_0 cannot be rejected. Sufficient evidence is not available to conclude that Holiday should change its production plan for the coming season.

(l) (Software) The p-value obtained is _________. With a level of significance of $\alpha = 0.05$, we cannot reject H_0 because $0.2822 > 0.05$.

(m) With $\alpha = 0.05$ and the t distribution with 24 degrees of freedom, ___________________ and ________________ are the critical values for the two-tailed test. The rejection rule using the test statistic is

__________________________________.

(n) Based on the test statistic $t = -1.10$, H_0 cannot be rejected. This result indicates that Holiday should continue its production planning for the coming season based on the expectation that _________.

Summary and Practical Advice

1. (Table 9.3) A summary of the hypothesis testing procedures about a population mean for the σ unknown case.

TABLE 9.3 Summary of Hypothesis Tests About a Population Mean: σ Unknown Case

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test Statistic	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $t \leq -t_\alpha$	Reject H_0 if $t \geq t_\alpha$	Reject H_0 if $t \leq -t_{\alpha/2}$ or if $t \geq t_{\alpha/2}$

2. The applicability of the hypothesis testing procedures of this section is dependent on the ____________________ being sampled from and the __________.

☺ **EXERCISES 9.4**: 23, 24, 27, 33

9.5 Population Proportion

1. Using p_0 to denote the hypothesized value for the population proportion, the three forms for a hypothesis test about a population proportion:

____

2. Hypothesis tests about a population proportion are based on the __________ between the sample proportion _____ and the hypothesized population proportion _____.

3. The ______________________ of p, the point estimator of the population parameter p, is the basis for developing the test statistic.

4. When the null hypothesis is true as an equality, the expected value of p equals the hypothesized value p_0; that is, ____________. The standard error of $\bar{p}$ is given by

   ____

5. (Recall, Chapter 7) we said that if __________ and ______________, the sampling distribution of $\bar{p}$ can be approximated by a __________ distribution. Under these conditions, which usually apply in practice, the quantity

   $$z = \qquad\qquad (9.3)$$

   ____

   has a standard normal probability distribution.

6. **Test Statistic for Hypothesis Tests About a Population Proportion**
   With ____________________, the standard normal random variable z is the test statistic used to conduct hypothesis tests about a population proportion.

   $$z =$$

   ____

7. $\boxed{\text{Example}}$ (Pine Creek golf course example). Over the past year, 20% of the players at Pine Creek were women. In an effort to increase the proportion of women players, Pine Creek implemented a special promotion designed to attract women golfers. One month after the promotion was implemented, the course manager requested a statistical study to determine whether the proportion of women players at Pine Creek had increased.

   (a) Because the objective of the study is to determine whether the proportion of women golfers increased, an upper tail test with _______________ is appropriate:

   ____

   (b) If H_0 can be rejected, the test results will give statistical support for the conclusion that the proportion of women golfers increased and the promotion was beneficial.

(c) The course manager specified that a level of significance of ____________ be used in carrying out this hypothesis test.

(d) The next step of the hypothesis testing procedure is to select a sample and compute the value of an appropriate test statistic. Suppose a random sample of $n = 400$ players was selected, and that $x = 100$ of the players were women. The proportion of women golfers in the sample is

____

Using equation (9.4), the value of the test statistic is

$$z =$$

____

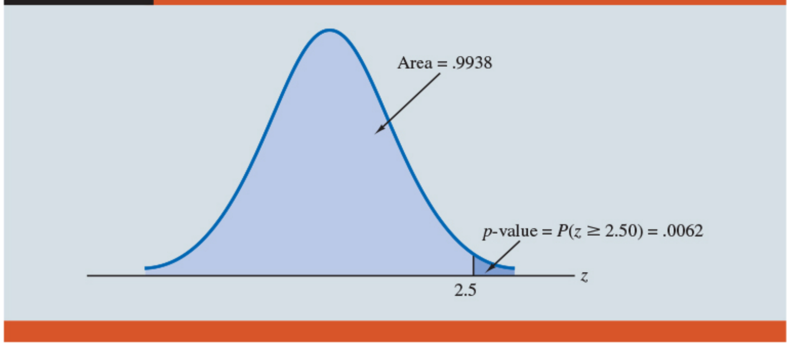
(e) *The p-value approach.*

i. The p-value is the probability that z is greater than or equal to ____________.
________________________, the p-value for the Pine Creek test is ________________________

ii. (Figure 9.7) Recall that the course manager specified a level of significance of $\alpha = 0.05$. A __________________________ gives sufficient statistical evidence to reject H_0 at the 0.05 level of significance.

iii. The test provides statistical support for the conclusion that the special promotion increased the proportion of women players at the Pine Creek golf course.

FIGURE 9.7 Calculation of the p-Value for the Pine Creek Hypothesis Test



(f) *The critical value approach.* The critical value corresponding to an area of 0.05 in the upper tail of a normal probability distribution is ________________.

Thus, the rejection rule using the critical value approach is to reject H_0 if ____________. Because __________________, H_0 is rejected.

(g) The p-value approach provides more information. With a p-value $= 0.0062$, the null hypothesis would be rejected for any level of significance __________________ __________________.

Summary

1. The procedure used to conduct a hypothesis test about a ________________________ is similar to the procedure used to conduct a hypothesis test about a ___________________.

2. Although we only illustrated how to conduct a hypothesis test about a population proportion for an upper tail test, similar procedures can be used for ____________ and __________________.

3. (Table 9.4) A summary of the hypothesis tests about a population proportion. We assume that $np \geq 5$ and $n(1-p) \geq 5$; thus the __________ probability distribution can be used to approximate the sampling distribution of $\bar{p}$.

TABLE 9.4 Summary of Hypothesis Tests About a Population Proportion

	Lower Tail Test	Upper Tail Test	Two-Tailed Test
Hypotheses	$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$
Test Statistic	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$
Rejection Rule: *p*-Value Approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$
Rejection Rule: Critical Value Approach	Reject H_0 if $z \leq -z_\alpha$	Reject H_0 if $z \geq z_\alpha$	Reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$

EXERCISES 9.5: 35, 36, 43, 44

9.6 Hypothesis Testing and Decision Making

1. The hypothesis testing applications are considered as ____________________:
   (a) formulate the null and alternative hypotheses, H_0, H_a.
   (b) specify the level of significance, α.
   (c) select a sample, $\mathbf{x} = \{x_1, x_2, \cdots, x_n\}$.
   (d) compute the value of a test statistic, $T(\mathbf{x})$.
   (e) compute the associated p-value.
   (f) compare the p-value to α.
   (g) conclude "reject H_0" and declare the results significant if p-value $\leq \alpha$; otherwise, we made the conclusion "do not reject H_0."
2. With a significance test, we control the probability of making the Type I error, but not the Type II error. Thus, we recommended the conclusion ____________________ rather than ________________ because the latter puts us at ________ of making the Type II error of accepting H_0 when it is false.
3. With the conclusion "do not reject H_0," the statistical evidence is considered ________________ and is usually an indication to ________________________________ until further research and testing can be undertaken.
4. If the purpose of a hypothesis test is to ____________________ when H_0 is true and a ____________________ when H_a is true, the decision maker may want to, and in some cases be forced to, take action with both the conclusion do not reject H_0 and the conclusion reject H_0. If this situation occurs, statisticians generally recommend controlling the probability of making a _________________.
5. With the probabilities of both the Type I and Type II error controlled, the conclusion from the hypothesis test is __________________________________.
6. $\boxed{\text{Example}}$ (lot-acceptance example) A quality control manager must decide to accept a shipment of batteries from a supplier or to return the shipment because of poor quality.

(a) Assume that design specifications require batteries from the supplier to have a mean useful life of at least 120 hours. To evaluate the quality of an incoming shipment, a sample of 36 batteries will be selected and tested.

(b) On the basis of the sample, a decision must be made to accept the shipment of batteries or to return it to the supplier because of poor quality.

(c) Let μ denote the mean number of hours of useful life for batteries in the shipment. The null and alternative hypotheses about the population mean:

____

i. If H_0 is rejected, the alternative hypothesis is concluded to be true. This conclusion indicates that the appropriate action is to __________ the shipment to the supplier.

ii. If H_0 is not rejected, the decision maker must still determine what action should be taken. Thus, without directly concluding that H_0 is true, but merely by not rejecting it, the decision maker will have made the decision to __________ the shipment as being of satisfactory quality.

(d) In such decision-making situations, it is recommended that the hypothesis testing procedure be extended to control the probability of making a Type II error.

7. Because a decision will be made and action taken when we do not reject H_0, knowledge of the probability of making a ________________ will be helpful.

9.7 Calculating The Probability of Type II Errors

1. $\boxed{\text{Example}}$ (lot-acceptance example) The null and alternative hypotheses about the mean number of hours of useful life for a shipment of batteries:

$$H_0 : \mu \geq 120, \quad H_a : \mu < 120.$$

(a) If H_0 is rejected, the decision will be to __________ the shipment to the supplier because the mean hours of useful life are less than the specified 120 hours.

(b) If H_0 is not rejected, the decision will be to __________ the shipment.

2. Suppose a level of significance of $\alpha = 0.05$, the test statistic in the σ known case:

$$z = \frac{\qquad}{\qquad}$$

3. The rejection rule for the lower tail test:

____

4. Suppose a sample of $n = 36$ batteries will be selected and based upon previous testing the population standard deviation can be assumed known with a value of $\sigma = 12$ hours.

5. The rejection rule indicates that we will reject H_0 if

$$z = \frac{\qquad}{\qquad}$$

6. Solving for $\bar{x}$ in the preceding expression indicates that we will reject H_0 if

____

7. Rejecting H_0 when $\bar{x} \leq 116.71$ means that we will make the decision to accept the shipment whenever $\bar{x} > 116.71$.

8. Compute probabilities associated with making a Type II error.

   (a) (Recall) we make a Type II error whenever the true shipment mean is less than 120 hours and we make the decision to accept $H_0 : \mu \geq 120$.

   (b) Hence, to compute the probability of making a Type II error, we must select a value of ______________________.

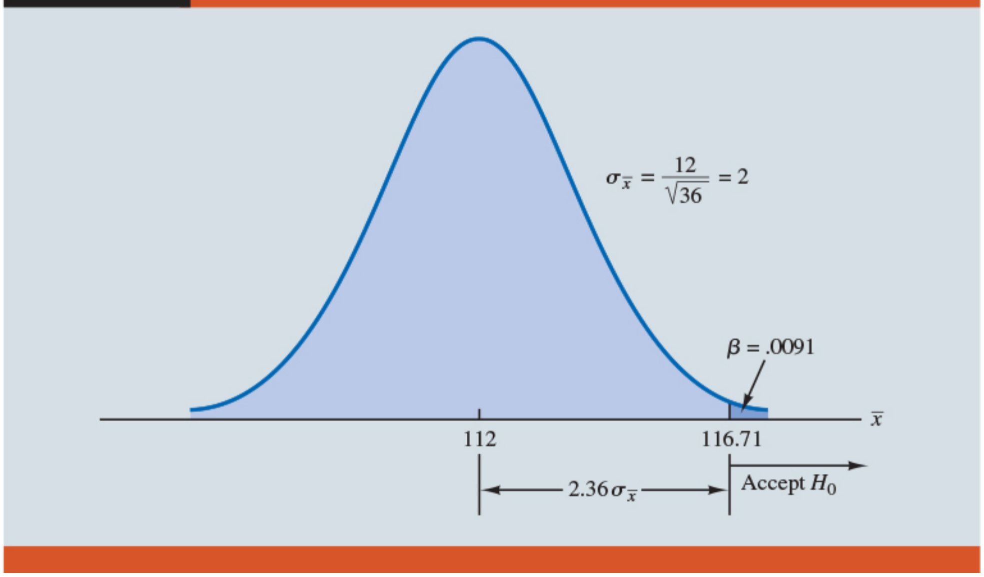
   (c) For example, suppose the shipment is considered to be of poor quality if the batteries have a mean life of $\mu = 112$ hours.

(d) If $\mu = 112$ is really true, what is the probability of accepting $H_0 : \mu \geq 120$ and hence committing a Type II error?

$$\underline{\hspace{6cm}}.$$

(e) (Figure 9.8) the sampling distribution of $\bar{x}$ when the mean is $\mu = 112$. The shaded area in the upper tail gives the probability of obtaining ______________.

FIGURE 9.8 Probability of a Type II Error When $\mu = 112$



(f) Using the standard normal distribution, we see that at $\bar{x} = 116.71$

$$z = \underline{\hspace{6cm}}$$

(g) The probability of making a Type II error when $\mu = 112$ is ____________________________.

(h) Therefore, we can conclude that if the mean of the population is 112 hours, the probability of making a Type II error is only 0.0091.

(i) We can repeat these calculations for other values of μ less than 120.

9. (Table 9.5) we show the probability of making a Type II error for a variety of values of μ less than 120. Note that as μ increases toward 120, the probability of making a Type II error increases toward an upper bound of 0.95. However, as μ decreases to values farther below 120, the probability of making a Type II error diminishes.

TABLE 9.5 Probability of Making a Type II Error for the Lot-Acceptance Hypothesis Test

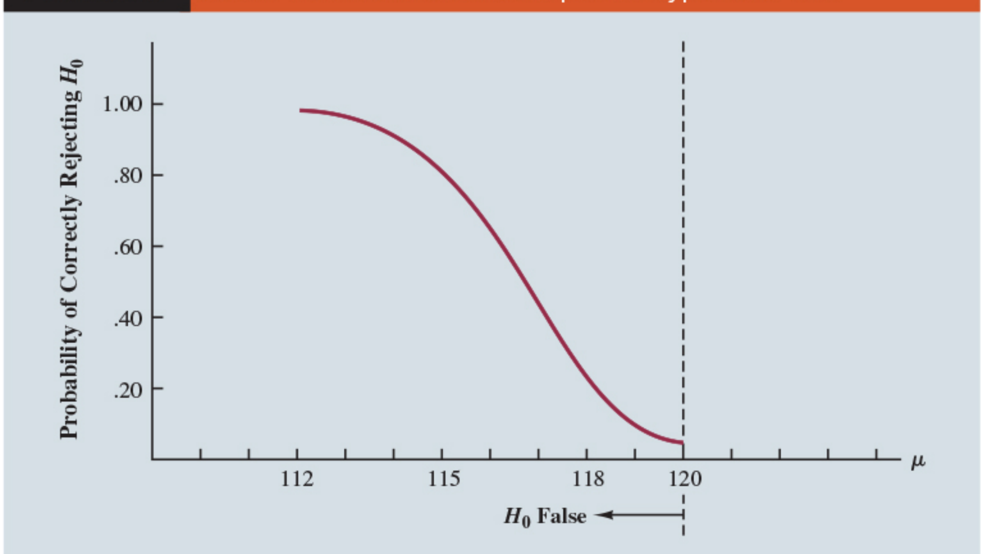
Value of μ	$z = \dfrac{116.71 - \mu}{12/\sqrt{36}}$	Probability of a Type II Error (β)	Power $(1 - \beta)$
112	2.36	.0091	.9909
114	1.36	.0869	.9131
115	.86	.1949	.8051
116.71	.00	.5000	.5000
117	−.15	.5596	.4404
118	−.65	.7422	.2578
119.999	−1.645	.9500	.0500

10. When the true population mean μ is ________________ the null hypothesis value of $\mu = 120$, the probability is __________ that we will make a Type II error.

11. For any particular value of μ, the ____________; that is, the probability of __________________ is 1 minus the probability of making a Type II error.

12. (Figure 9.9) *Power curve*: the power associated with each value of μ:

FIGURE 9.9 Power Curve for the Lot-Acceptance Hypothesis Test



(a) Note that the power curve extends over the values of μ for which the __________.

(b) The __________ of the power curve at any value of μ indicates the probability of correctly rejecting H_0 when H_0 is false.

13. **The step-by-step procedure to compute the probability of making a Type II error in hypothesis tests about a population mean**

    (a) Formulate the null and alternative hypotheses.

    (b) Use the level of significance α and the critical value approach to determine the critical value and the ________________ for the test.

    (c) Use the rejection rule to solve for the value of the ________________ corresponding to the critical value of the test statistic.

    (d) Use the results from step (c) to state the values of the sample mean that lead to the acceptance of H_0. These values define the ____________________ for the test.

    (e) Use the sampling distribution of $\bar{x}$ for a value of μ satisfying H_a, and the acceptance region from step (d), to compute the probability that the sample mean will be in the acceptance region.

    (f) This probability is the probability of making a Type II error at the chosen value of μ.

😍 **EXERCISES 9.7**: 46, 50, 53

9.8 Determining The Sample Size for a Hypothesis Test about a Population Mean

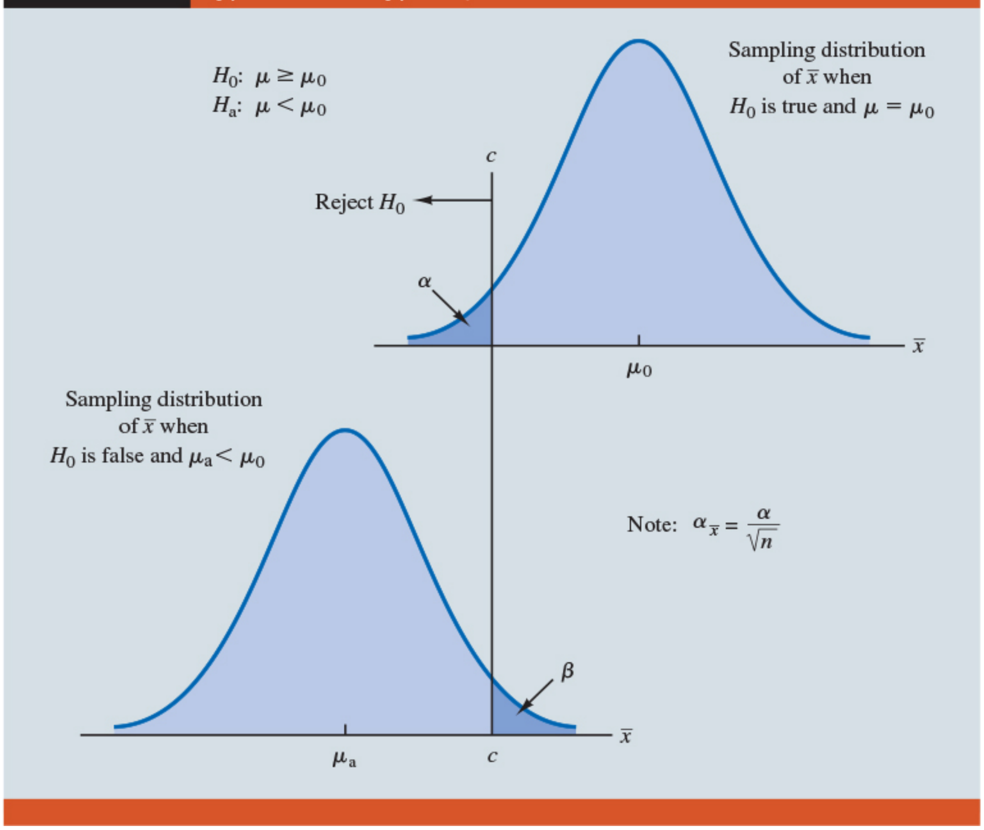
1. Assume that a hypothesis test is to be conducted about the value of a population mean. The level of significance specified by the user determines the probability of making a Type I error for the test. By controlling the ______________, the user can also control the probability of making a ________________.

2. Let us show how a sample size can be determined for the following lower tail test about a population mean.

 ____

3. (Figure 9.10) The upper panel is the sampling distribution of $\bar{x}$ when H_0 is true with $\mu = \mu_0$.

FIGURE 9.10 Determining the Sample Size for Specified Levels of the Type I (α) and Type II (β) Errors



4. For a lower tail test, the critical value of the test statistic is denoted ________. In the upper panel of the figure the vertical line, ____________, is the corresponding value of ______.

5. If we reject H_0 when $\bar{x} \leq c$, the probability of a Type I error will be α: ________________________.

6. With z_α representing the z value corresponding to an area of α in the ____________ of the standard normal distribution, we compute c using the following formula:

$$c = \underline{\hspace{4cm}}$$

7. (Figure 9.10) The lower panel is the sampling distribution of $\bar{x}$ when the alternative hypothesis is true with ________________. The shaded region shows ______, the probability of a Type II error that the decision maker will be exposed to if the null hypothesis is accepted when $\bar{x} > c$.

8. With z_β representing the z value corresponding to an area of β in the upper tail of the standard normal distribution, we compute c using the following formula:

$$c = \underline{\hspace{4cm}} \tag{9.6}$$

9. Now what we want to do is to select a value for c so that when we _____________ and ______________, the probability of a Type I error is equal to the chosen value of ______ and the probability of a Type II error is equal to the chosen value of ______. Therefore, both equations (9.5) and (9.6) must provide the same value for c:

$$\begin{aligned}
\mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}} &= \mu_a + z_\beta \frac{\sigma}{\sqrt{n}} \\
\mu_0 - \mu_a &= z_\alpha \frac{\sigma}{\sqrt{n}} + z_\beta \frac{\sigma}{\sqrt{n}} \\
\mu_0 - \mu_a &= \frac{(z_\alpha + z_\beta)\sigma}{\sqrt{n}} \\
\sqrt{n} &= \frac{(z_\alpha + z_\beta)\sigma}{(\mu_0 - \mu_a)}
\end{aligned}$$

10. **Sample Size for a One-Tailed Hypothesis Test About a Population Mean**

$$n = \underline{\hspace{4cm}} \tag{9.7}$$

where

- $z_\alpha = z$ value providing an area of α in the upper tail of a standard normal distribution.
- $z_\beta = z$ value providing an area of β in the upper tail of a standard normal distribution.
- $\sigma =$ the population standard deviation.
- $\mu_0 =$ the value of the population mean in the null hypothesis.
- $\mu_\alpha =$ the value of the population mean used for the Type II error.

Note: In a two-tailed hypothesis test, use (9.7) with ________ replacing ________.

11. Example lot-acceptance example

   (a) The design specification for the shipment of batteries indicated a mean useful life of at least 120 hours for the batteries. Shipments were rejected if $H_0 : \mu \geq 120$ was rejected.

   (b) Let us assume that the quality control manager makes the following statements about the allowable probabilities for the Type I and Type II errors.

      i. *Type I error statement*: If the mean life of the batteries in the shipment is $\mu = 120$, I am willing to risk an $\alpha = 0.05$ probability of rejecting the shipment.

      ii. *Type II error statement*: If the mean life of the batteries in the shipment is 5 hours under the specification (i.e., $\mu = 115$), I am willing to risk a $\beta = 0.10$ probability of accepting the shipment.

      iii. These statements are based on the judgment of the manager. Someone else might specify different restrictions on the probabilities. However, statements about the allowable probabilities of both errors must be made before the sample size can be determined.

   (c) In the example, ____________ and ____________. Using the standard normal probability distribution, we have ______________ and ______________. From the statements about the error probabilities, we note that ____________ and ____________. Finally, the population standard deviation was assumed known at __________.

   (d) The recommended sample size for the lot-acceptance example is

$$n = \frac{\qquad\qquad}{\qquad\qquad} = \frac{\qquad\qquad\qquad}{\qquad\qquad\qquad}$$

   Rounding up, we recommend a sample size of 50.

   (e) Because both the Type I and Type II error probabilities have been ____________ at allowable levels with $n = 50$, the quality control manager is now justified in using the accept H_0 and reject H_0 statements for the hypothesis test.

12. We can make three observations about the relationship among α, β, and the sample size n.

(a) Once two of the three values are known, the other can be computed.

(b) For a given α, increasing ______ will reduce ______.

(c) For a given n, decreasing ______ will increase ______, whereas increasing ______ will decrease ______.

13. The third observation should be kept in mind when the probability of a Type II error is not being controlled. It suggests that one should not choose ____________________________ for the level of significance ______.

14. For a given sample size, choosing a smaller level of significance means more exposure to a Type II error. Inexperienced users of hypothesis testing often think that smaller values of α are always better. They are better if we are concerned only about making a Type I error. However, smaller values of α have the disadvantage of increasing the probability of making a ________________.

😍 **EXERCISES 9.8**: 55, 59

9.9 Big Data And Hypothesis Testing*

😍 **SUPPLEMENTARY EXERCISES**: 69, 79, 83