統計學 (二)

Anderson's Statistics for Business & Economics (14/E)

Chapter 13: Experimental Design and Analysis of Variance

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Overview

1. The statistical studies can be classified as either ____________ or ____________.
2. In an experimental statistical study, an experiment is conducted to generate the data.
   (a) An experiment begins with identifying a ____________ of interest.
   (b) Then one or more other variables, thought to be ____________, are identified and ____________, and
   (c) data are collected about how those variables ____________ the variable of interest.
3. In an observational study, data are usually obtained through sample ____________ and not a controlled experiment.
4. Good design principles are still employed, but the ____________ controls associated with an experimental statistical study are often not possible.

5. $\boxed{\text{Example}}$ In a study of the relationship between smoking and lung cancer the researcher cannot assign a smoking habit to subjects. The researcher is restricted to simply observing the effects of smoking on people who already smoke and the effects of not smoking on people who do not already smoke.

6. In this chapter we introduce three types of experimental designs: a ____________ ____________ design, a ____________ design*, and a ____________ experiment*.

7. Analysis of variance (____________) can analyze the results of regression studies involving both experimental and observational data.

13.1 An Introduction to Experimental Design and Analysis of Variance

1. $\boxed{\text{Example}}$ Chemitech Inc. developed a new filtration system for municipal water supplies.

   (a) The industrial engineering group is responsible for determining the best assembly method (method A, method B, and method C) for the new filtration system.

   (b) Managers at Chemitech want to determine which assembly method can produce the greatest number of filtration systems per week.

   (c) In the Chemitech experiment, ____________ is the ____________ variable or ____________.

   (d) Because three assembly methods correspond to this factor, we say that three ____________ are associated with this experiment; each treatment corresponds to one of the three assembly methods.

2. (**single-factor experiment**) The Chemitech problem is an example of a ____________ experiment; it involves one ____________ factor (method of assembly).

3. More complex experiments may consist of __________ factors; some factors may be categorical and others may be quantitative.

4. (**populations**) The three assembly methods or treatments define the three __________ of interest for the Chemitech experiment. One population is all Chemitech employees who use assembly method A, another is those who use method B, and the third is those who use method C.

5. (**objective**) Note that for each population the __________ or __________ variable is the __________ of filtration systems assembled per week, and the primary statistical objective of the experiment is to determine whether the __________ __________ produced per week is the same for all three populations (methods).

6. (**experimental units**) Suppose a random sample of three employees is selected from all assembly workers at the Chemitech production facility. In experimental design terminology, the three randomly selected __________ are the experimental __________.

7. (**completely randomized design**) A __________ requires that each of the three assembly methods or treatments be assigned randomly to one of the experimental units or workers.

   (a) For example, method A might be randomly assigned to the second worker, method B to the first worker, and method C to the third worker.

   (b) Note that this experiment would result in only one measurement or number of units assembled for each treatment.

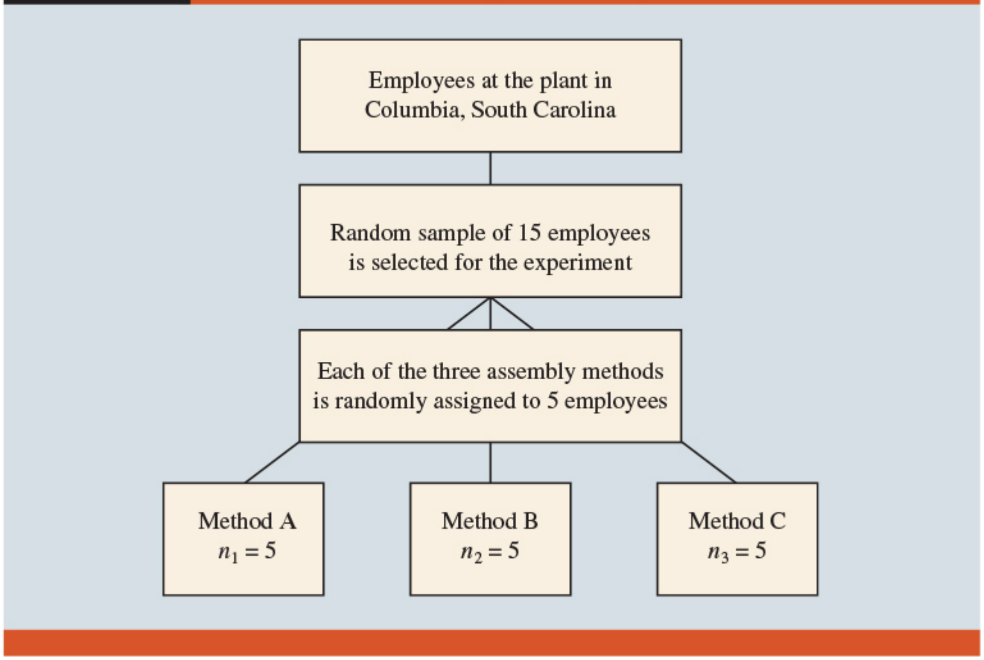
8. (**replicates**) To obtain additional data for each assembly method, we must __________ or __________ the basic experimental process.

   (a) Suppose, for example, we selected 15 workers and then randomly as signed each of the three treatments to 5 of the workers.

   (b) Because each method of assembly is assigned to 5 workers, we say that __________ have been obtained.

(Figure 13.1) the completely randomized design for the Chemitech experiment.

FIGURE 13.1 Completely Randomized Design for Evaluating the Chemitech Assembly Method Experiment



Data Collection

1. Once we are satisfied with the experimental design, we proceed by collecting and analyzing the data. In the Chemitech case, the employees would be instructed in how to perform the assembly method assigned to them and then would begin assembling the new filtration systems using that method.

2. (Table 13.1) After this assignment and training, the number of units assembled by each employee during one week is as shown in Table 13.1. The sample means, sample variances, and sample standard deviations for each assembly method are also provided. From these data, ______________ appears to result in higher production rates than either of the other methods.

TABLE 13.1 Number of Units Produced by 15 Workers

	Method A	Method B	Method C
	58	58	48
	64	69	57
	55	71	59
	66	64	47
	67	68	49
Sample mean	62	66	52
Sample variance	27.5	26.5	31.0
Sample standard deviation	5.244	5.148	5.568

3. (**question**) is whether the three sample means observed are different enough for us to conclude that the means of the populations corresponding to the three methods of assembly are different.

4. Turn the question to Statistical terms: μ_1, μ_2, μ_3 = mean number of units produced per week using method A, B, C, respectively

5. Although we will never know the actual values of μ_1, μ_2, and μ_3, we want to use the sample means to test the following hypotheses.

$$H_0 : \mu_1 = \mu_2 = \mu_3, \quad H_a : \text{Not all population means are equal}$$

6. The ______________________ is the statistical procedure used to determine whether the observed differences in the three sample means are large enough to reject H_0.

Assumptions for Analysis of Variance

Three assumptions are required to use analysis of variance.

1. For each population, the response variable is ________________.
   Implication: In the Chemitech experiment, the number of units produced per week (response variable) must be normally distributed for each assembly method.

2. The variance of the ________________, is the same for all of the populations.

Implication: In the Chemitech experiment, the variance of the number of units produced per week must be the same for each assembly method.

3. The observations must be ____________.
   Implication: In the Chemitech experiment, the number of units produced per week for each employee must be independent of the number of units produced per week for any other employee.

Analysis of Variance: A Conceptual Overview

1. If the means for the three populations are equal, we would expect the three ____________ to be close together.

   (a) The more the sample means ________, the stronger the evidence we have for the conclusion that the population means ________.

   (b) If the __________ among the sample means is __________ it supports ______; if the variability among the sample means is ________," it supports ______.

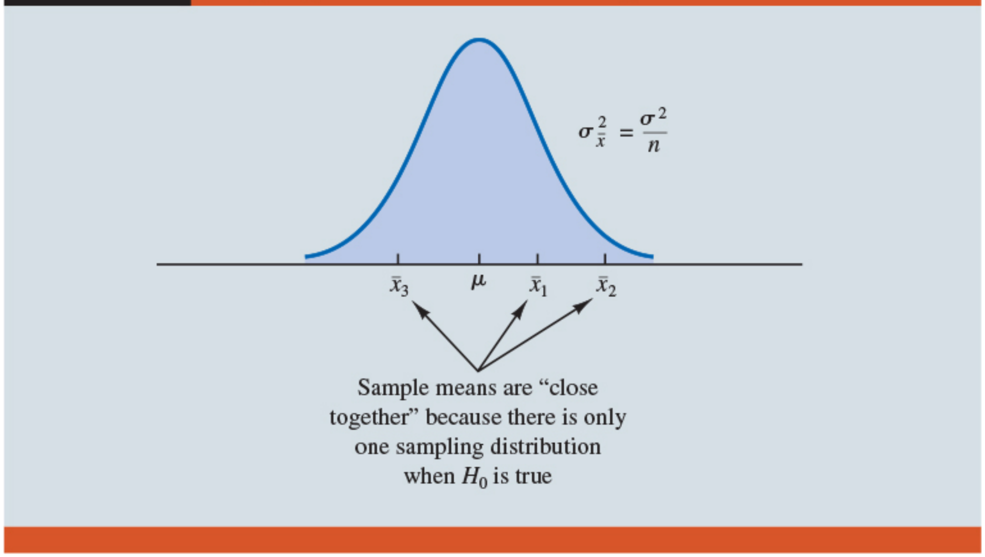
2. If the null hypothesis, $H_0 : \mu_1 = \mu_2 = \mu_3$, is true, we can use the ______________ the sample means to develop an estimate of ______.

   (a) If the assumptions for analysis of variance are satisfied and the null hypothesis is true, each sample will have come from the same __________ distribution with mean _____ and variance ______.

   (b) (Chapter 7) the sampling distribution of the sample mean $\bar{x}$ for a simple random sample of size n from a normal population will be normally distributed with mean _____ and variance ________. (________________)

   (c) (Figure 13.2) if H_0 is true, we can think of each of the three sample means, $\bar{x}_1 = 62, \bar{x}_2 = 66$, and $\bar{x}_3 = 52$ from Table 13.1, as values drawn at random from the sampling distribution shown in Figure 13.2.

FIGURE 13.2 Sampling Distribution of $\bar{x}$ Given H_0 Is True



3. When the sample sizes are equal, as in the Chemitech experiment, the best estimate of the mean of the sampling distribution of $\bar{x}$ is the __________ or ________________ ________________. In the Chemitech experiment, an estimate of the mean of the sampling distribution of $\bar{x}$ is ______________________. We refer to this estimate as the ____________________.

4. An estimate of the variance of the sampling distribution of $\bar{x}$, ______, is provided by the variance of the three sample means.

$$s_{\bar{x}}^2 = \underline{\hspace{8cm}}$$

5. Because ____________, solving for σ^2 gives

$$\sigma^2 = \underline{\hspace{2cm}}$$

Hence,

$$\text{Estimate of } \sigma^2 = n\ (\text{Estimate of } \sigma_{\bar{x}}^2) = \underline{\hspace{4cm}}.$$

6. The result, $ns_{\bar{x}}^2 = 260$, is referred to as the ____________________ estimate of σ^2.

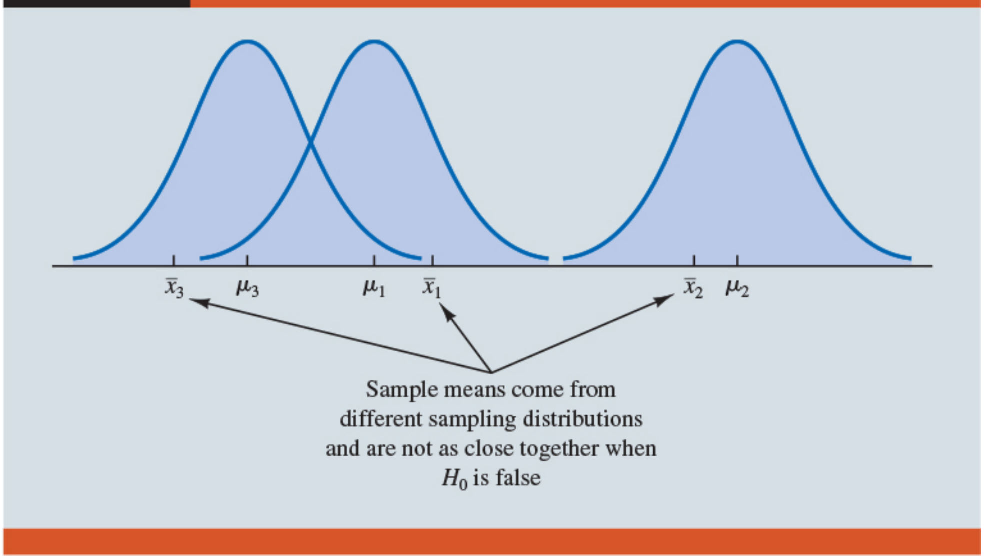
7. The between treatments estimate of σ^2 is based on the assumption that ____________. In this case, each sample comes from the ________ population, and there is only _______ sampling distribution of $\bar{x}$.

8. Illustrate what happens when H_0 is false, suppose the population means all __________.

   (a) Note that because the three samples are from __________ populations with different means, they will result in three __________ sampling distributions.

   (b) (Figure 13.3) The sample means are not as close together as they were when H_0 was true. Thus, $s_{\bar{x}}^2$ will be larger, causing the between treatments estimate of σ^2 to be __________.

FIGURE 13.3 Sampling Distributions of $\bar{x}$ Given H_0 Is False



   (c) In general, when the population means are not equal, the between treatments estimate will __________ the population variance σ^2.

   (d) When a simple random sample is selected from each population, each of the sample variances provides an __________ estimate of σ^2. Hence, we can __________ or __________ the individual estimates of σ^2 into one overall estimate.

   (e) The estimate of σ^2 obtained in this way is called the __________ or __________ estimate of σ^2.

   (f) Because each sample variance provides an estimate of σ^2 based only on the variation within each sample, the within treatments estimate of σ^2 is not affected by whether the population means are equal.

   (g) When the sample sizes are equal, the within treatments estimate of σ^2 can be obtained by computing the __________ of the individual sample variances.

9. $\boxed{\text{Example}}$ For the Chemitech experiment we obtain

   Within treatments estimate of $\sigma^2 =$ ____________________________

   (a) The between treatments estimate of σ^2 (260) is much __________ than the within treatments estimate of σ^2 (28.33).

   (b) The _________ of these two estimates is $260/28.33 = 9.18$.

10. If the null hypothesis is _________,

    (a) The between treatments approach provides a _________ estimate of σ^2.

    (b) The two estimates will be similar and their ratio will be close to ______.

11. If the null hypothesis is _________,

    (a) The between treatments approach__________________ σ^2

    (b) the between treatments estimate will be larger than the within treatments estimate, and their ratio will be _________.

12. In the next section we will show how large this ratio must be to reject H_0.

13. **Summary**: The logic behind ANOVA is based on the development of two independent estimates of the common population variance _______.

    (a) One estimate of σ^2 is based on the variability ___________ the sample means themselves.

    (b) The other estimate of σ^2 is based on the variability of the data ___________ each sample.

    (c) By comparing these two estimates of σ^2, we will be able to determine whether the population means are equal.

補充說明:

ANOVA Table

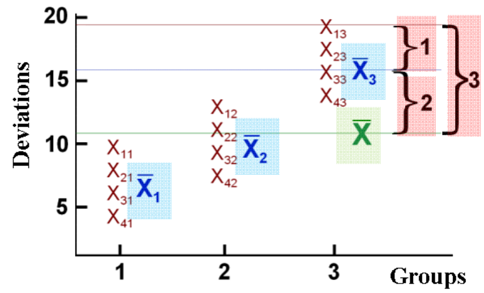
Groups

$$
\begin{array}{c|c|c|c|c}
1 & 2 & \cdots\ j\ \cdots & & k \\
\hline
X_{11} & X_{12} & \cdots\ X_{1j}\ \cdots & & X_{1k} \\
X_{21} & X_{22} & \cdots\ X_{2j}\ \cdots & & X_{2k} \\
 & & \cdots & & \\
X_{i1} & X_{i2} & \cdots\ X_{ij}\ \cdots & & X_{ik} \\
\vdots & & & & X_{n_k k} \\
 & X_{n_2 2} & \cdots\ \vdots\ \cdots & & \\
X_{n_1 1} & & X_{n_j j} & &
\end{array}
$$

$$T_j = \sum_{i=1}^{n_j} X_{ij} \qquad \bar{X}_j = \frac{T_j}{n_j}$$

$$T = \sum_{j=1}^{k} T_j \qquad \bar{X} = \frac{T}{N}$$

$$S^2 = \sum_{j=1}^{k}\sum_{i=1}^{n_j} \frac{(X_{ij}-\bar{X})^2}{N-1}$$



$$(X_{ij}-\bar{X}) = (X_{ij}-\bar{X}_j) + (\bar{X}_j-\bar{X})$$

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$$

$$X_{ij} = \mu_j + \epsilon_{ij} \qquad i = 1,\cdots,n_j \quad j = 1,\cdots,k$$

$$\epsilon_{ij} \sim N(0,\sigma^2)$$

$$\sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X})^2 = \sum_{j=1}^{k}\sum_{i=1}^{n_j}[(X_{ij}-\bar{X}_j)+(\bar{X}_j-\bar{X})]^2$$

$$\sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X})^2 = \sum_{j=1}^{k}\sum_{i=1}^{n_j}(X_{ij}-\bar{X}_j)^2 + \sum_{j=1}^{k}\sum_{i=1}^{n_j}(\bar{X}_j-\bar{X})^2$$

ANOVA Table

Source	SS	df	MS	F	p
Between	SS_B	$k-1$	MS_B	MS_B/MS_W	< 0.05
Within	SS_W	$N-k$	MS_W		
Total	SS_T	$N-1$			

$$SS_{Total} = SS_{Within} + SS_{Between}$$

$$F = \frac{MS_{Between}}{MS_{Within}}$$

Reject H_0, if $F_{obs} > F_{\{\alpha,k-1,N-k\}}$

13.2 Analysis of Variance and the Completely Randomized Design

1. How analysis of variance can be used to test for the equality of k population means for a ____________ randomized design.

2. The general form of the hypotheses tested is

$$H_0: \underline{\hspace{4cm}}, \quad H_a: \text{Not all population means are equal}$$

where μ_j is mean of the jth population.

3. We assume that a simple random sample of size ______ has been selected from each of the k ______________ or ______________.

4. For the resulting sample data, let

   (a) x_{ij}: value of observation i for treatment j, $i = 1, 2, \cdots, n_j$, $j = 1, 2, \cdots, k$
   (b) n_j: number of observations for treatment j.
   (c) $\bar{x}_j$: sample mean for treatment j, ______________.
   (d) s_j^2: sample variance for treatment j, ______________
   (e) s_j: sample standard deviation for treatment j

5. The overall sample mean, denoted ______, is the sum of all the observations divided by the total number of observations:

$$\bar{\bar{x}} = \underline{\hspace{4cm}} \qquad (13.3)$$

where $n_T = n_1 + n_2 + \cdots + n_k$ (13.4).

6. If the size of each sample is n, $n_T = kn$; the overall sample mean is just the __________ of the k sample means.

$$\bar{\bar{x}} = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}}{kn} = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}/n}{k} = \underline{\hspace{3cm}} \qquad (13.5)$$

7. The overall sample mean can also be computed as a __________________ of the k sample means.

$$\bar{\bar{x}} = \underline{\hspace{5cm}}$$

8. $\boxed{\text{Example}}$ Each sample in the Chemitech experiment consists of $n = 5$ observations (Table 13.1), we obtained the following result:

$$\bar{\bar{x}} = \frac{62 + 66 + 52}{3} = 60$$

If the null hypothesis is true ($\mu_1 = \mu_2 = \mu_3 = \mu$), the overall sample mean of 60 is the ________ estimate of the population mean μ.

Between-Treatments Estimate of Population Variance

1. A between treatments estimate of σ^2 when the sample sizes were equal.

   (a) This estimate of σ^2 is called the ________________ due to ________________ and is denoted ____________:

   $$MSTR = \frac{}{} \qquad (13.6)$$

   (b) The numerator in equation (13.6) is called the ________________ due to treatments and is denoted __________.

   (c) The denominator, $k-1$, represents the degrees of freedom associated with SSTR.

   (d) **Mean Square Due to Treatments**

   $$MSTR = \frac{SSTR}{k-1} \qquad (13.7)$$

   where

   $$SSTR = \frac{}{} \qquad (13.8)$$

   (e) If H_0 is true, MSTR provides an ____________ estimate of σ^2. However, if the means of the k populations are not equal, MSTR is not an unbiased estimate of σ^2; in fact, in that case, MSTR should ________________ σ^2.

   (f) If each sample consists of n observations, equation (13.6) can be written as

   $$MSTR = \frac{}{} = \frac{}{} = \underline{}$$

2. Example For the Chemitech data in Table 13.1, we obtain the following results:

$$SSTR = \sum_{j=1}^{k} n_j(\bar{x}_j - \bar{\bar{x}})^2 = 5(62-60)^2 + 5(66-60)^2 + 5(52-60)^2 = \underline{\qquad\qquad}$$

$$MSTR = \frac{SSTR}{k-1} = \frac{520}{2} = \underline{\qquad\qquad}$$

Within-Treatments Estimate of Population Variance

1. A within treatments estimate of σ^2 when the sample sizes were equal.

   (a) This estimate of σ^2 is called the ________________ due to _________ and is denoted _________:

   $$MSE = \frac{\qquad\qquad}{\qquad\qquad\qquad\qquad} \qquad (13.9)$$

   (b) The numerator in equation (13.9) is called the __________________ due to error and is denoted _________.

   (c) The denominator of MSE is referred to as the degrees of freedom associated with SSE.

   (d) **Mean Square Due to Error**

   $$MSE = \frac{\qquad\qquad}{\qquad\qquad\qquad} \qquad (13.10)$$

   $$\text{where } SSE = \frac{\qquad\qquad}{\qquad\qquad\qquad\qquad} \qquad (13.11)$$

   (e) Note that MSE is based on the variation within each of the treatments; it is not influenced by whether the null hypothesis is true. Thus, MSE ___________ provides an _____________ estimate of σ^2.

   (f) If each sample has n observations, $n_T = kn$; thus, ____________________, and equation (13.9) can be rewritten as

   $$MSE = \frac{\sum_{j=1}^{k}(n-1)s_j^2}{k(n-1)} = \underline{\qquad\qquad\qquad}$$

   (g) If the sample sizes are the same, MSE is the average of the ______________________.

   (h) Note that it is the same result we used in Section 13.1 when we introduced the concept of the within-treatments estimate of σ^2.

2. Example For the Chemitech data in Table 13.1 we obtain the following results.

$$\begin{aligned} SSE &= \sum_{j=1}^{k}(n_j-1)s_j^2 = (5-1)27.5 + (5-1)26.5 + (5-1)31 = \underline{\hspace{2cm}} \\ MSE &= \frac{SSE}{n_T-k} = \frac{340}{15-3} = \frac{340}{12} = \underline{\hspace{2cm}} \end{aligned}$$

Comparing the Variance Estimates: The F Test

1. If the null hypothesis is ________, MSTR and MSE provide two independent, unbiased estimates of σ^2.

2. (Chapter 11) For ___________ populations, the sampling distribution of the ratio of two independent estimates of σ^2 follows an ______ distribution.

3. Hence, if the null hypothesis is true and the ANOVA assumptions are valid, the sampling distribution of _________________ is an ______ distribution with numerator degrees of freedom equal to ________ and denominator degrees of freedom equal to _________.
   (a) If the null hypothesis is true, the value of $MSTR/MSE$ should appear to have been selected from this ______ distribution.
   (b) If the null hypothesis is false, the value of MSTR/MSE will be ___________ because MSTR overestimates σ^2.

4. Hence, we will reject H_0 if the resulting value of $MSTR/MSE$ appears to be ____________ to have been selected from an F distribution with $k-1$ numerator degrees of freedom and n_T-k denominator degrees of freedom.

5. **Test Statistic for the Equality of K Population Means**

$$\underline{\hspace{4cm}} \qquad (13.12)$$

6. The test statistic follows an F distribution with $k-1$ degrees of freedom in the numerator and n_T-k degrees of freedom in the denominator. (_________________)

7. Example Let us return to the Chemitech experiment and use a level of significance ____________ to conduct the hypothesis test.

(a) The value of the test statistic is

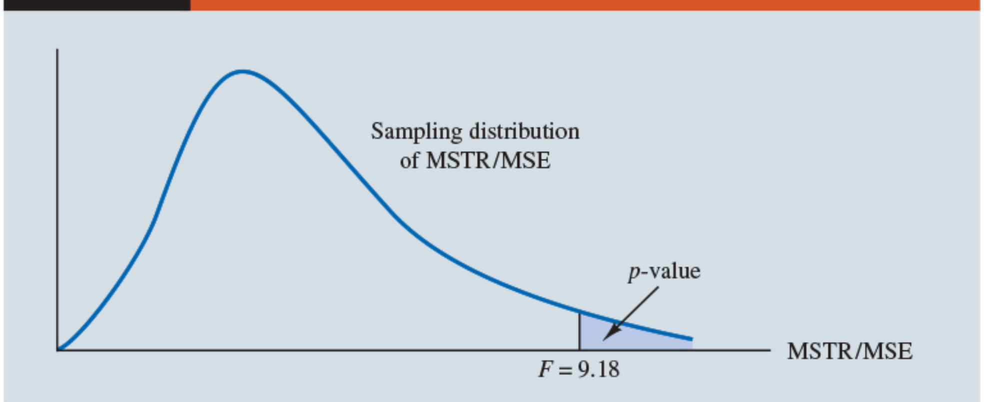
$$F = \frac{MSTR}{MSE} = \underline{\hspace{3cm}}$$

(b) The numerator degrees of freedom is $k-1 = 3-1 = 2$ and the denominator degrees of freedom is $n_T-k = 15-3 = 12$.

(c) Because we will only reject the null hypothesis for large values of the test statistic, the p-value is the ________________ of the F distribution to the right of the test statistic $F = 9.18$.

(d) (Figure 13.4) shows the sampling distribution of $F = MSTR/MSE$, the value of the test statistic, and the upper tail area that is the p-value for the hypothesis test.

FIGURE 13.4 Computation of *p*-Value Using the Sampling Distribution of MSTR/MSE



(e) (Table 4 of Appendix B) the upper tail of an F distribution with 2 numerator degrees of freedom and 12 denominator degrees of freedom.

Area in Upper Tail	0.10	0.05	0.025	0.01
F Value ($df_1 = 2$, $df_2 = 12$)	2.81	3.89	5.10	6.93

(f) (**the p-value approach**) Because ________________, the area in the upper tail at $F = 9.18$ is less than 0.01. Thus, the p-value is less than 0.01 (Software: p-value $= 0.004$.)

(g) With ________________, H_0 is rejected.

(h) (**conclusion**) The test provides sufficient evidence to conclude that the means of the three populations are not equal. In other words, analysis of variance supports the conclusion that the population mean number of units produced per week for the three assembly methods are not equal.

(i) (**the critical value approach**) With ______________________, and conclude that the means of the three populations are not equal.

8. **Test for the Equality of K Population Means**

$$H_0: \quad \mu_1 = \mu_2 = \cdots = \mu_k$$
$$H_a: \quad \text{Not all population means are equal}$$

Test Statistic

$$F = \frac{MSTR}{MSE}$$

Rejection Rule

$$p\text{-value approach} \quad : \quad \text{Reject } H_0 \text{ if } p\text{-value} \leq \alpha$$
$$\text{Critical value approach} \quad : \quad \text{Reject } H_0 \text{ if } F \geq F_{\alpha.k-1,n_T-k}$$

ANOVA Table

1. (Table 13.2) The results of the preceding calculations can be displayed conveniently in a table referred to as the analysis of variance or __________ table. The general form of the ANOVA table for a completely randomized design is:

TABLE 13.2 ANOVA Table for a Completely Randomized Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F	p-value
Treatments	SSTR	$k-1$	$\text{MSTR} = \frac{\text{SSTR}}{k-1}$	$\frac{\text{MSTR}}{\text{MSE}}$	
Error	SSE	$n_T - k$	$\text{MSE} = \frac{\text{SSE}}{n_T - k}$		
Total	SST	$n_T - 1$			

2. (Table 13.3) JMP/Excel output

TABLE 13.3 Analysis of Variance Table for the Chemitech Experiment

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	*F*	*p*-value
Treatments	520	2	260.00	9.18	.004
Error	340	12	28.33		
Total	860	14			

3. Total sum of squares (SST):

   (a) The sum of squares associated with the source of variation referred to as __________ is called the total sum of squares (________).

   (b) $SST =$ _______________, and that the degrees of freedom associated with this ________ sum of squares is the sum of the degrees of freedom associated with the sum of squares due to ____________ and the sum of squares due to ________. (________________________.)

   (c) We point out that SST divided by its degrees of freedom $n_T - 1$ is the ____________________ that would be obtained if we treated the entire set of 15 observations as one data set.

   (d) With the entire data set as one sample, the formula for computing the total sum of squares, SST, is

$$SST = \qquad\qquad\qquad\qquad \text{(13.13)}$$

4. ANOVA can be viewed as the process of partitioning the total sum of squares and the degrees of freedom into their corresponding sources: ____________________.

Computer Results for Analysis of Variance

1. (Figure 13.5) JMP/Excel output for the Chemitech experiment:

FIGURE 13.5 Output for the Chemitech Experiment Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	2	520.0	260.00	9.18	.004
Error	12	340.0	28.33		
Total	14	860.0			

Model Summary

S	R-sq	R-sq (adj)
5.32291	60.47%	53.88%

Means

Factor	N	Mean	StDev	95% CI
Method A	5	62.00	5.24	(56.81, 67.19)
Method B	5	66.00	5.15	(60.81, 71.19)
Method C	5	52.00	5.57	(46.81, 57.19)

Pooled StDev = 5.32291

2. The square root of MSE provides the best estimate of the population standard deviation σ. This estimate of σ in Figure 13.5 is Pooled StDev; it is equal to 5.323.

3. A 95% confidence interval estimate of the population mean for Method A.

$$ \qquad (13.15)$$

   where s is the estimate of the population standard deviation σ. Because the best estimate of σ is provided by the Pooled StDev, we use a value of 5.323 for σ in expression (13.15).

4. The degrees of freedom for the t value is 12, the degrees of freedom associated with the error sum of squares. Hence, with $t_{0.025} = 2.179$ we obtain

$$62 \pm 2.179 \frac{5.323}{\sqrt{5}} = 62 \pm 5.19$$

   Thus, the individual 95% confidence interval for Method A goes from $62 - 5.19 = 56.81$ to $62 + 5.19 = 67.19$.

Testing for the Equality of k Population Means: An Observational Study

1. ANOVA can also be used to test for the equality of three or more population means using data obtained from an ____________________.

2. Example (Table 13.4) National Computer Products, Inc. (NCP) manufactures printers and fax machines at plants located in Atlanta, Dallas, and Seattle. To measure how much employees at these plants know about quality management, a random sample of 6 employees was selected from each plant and the employees selected were given a quality awareness examination. The examination scores for these 18 employees are shown in Table 13.4. Managers want to use these data to test the hypothesis that the mean examination score is the same for all three plants.

TABLE 13.4 Examination Scores for 18 Employees

	Plant 1	Plant 2	Plant 3
	Atlanta	**Dallas**	**Seattle**
	85	71	59
	75	75	64
	82	73	62
	76	74	69
	71	69	75
	85	82	67
Sample mean	79	74	66
Sample variance	34	20	32
Sample standard deviation	5.83	4.47	5.66

3. Define population 1 as all employees at the Atlanta plant, population 2 as all employees at the Dallas plant, and population 3 as all employees at the Seattle plant. Let _______ mean examination score for population $j, j = 1, 2, 3$

4. Want to use the sample results to test the following hypotheses:

$$H_0: \quad \mu_1 = \mu_2 = \mu_3$$
$$H_a: \quad \text{Not all population means are equal}$$

5. Note that the hypothesis test for the NCP observational study is ____________________ as the hypothesis test for the Chemitech experiment.

6. Even though the same ANOVA methodology is used for the analysis, it is worth noting how the NCP observational statistical study differs from the Chemitech experimental statistical study.

7. The individuals who conducted the NCP study had ____________ over how the plants were assigned to individual employees. That is, the plants were already in operation and a particular employee worked at one of the three plants. All that NCP could do was to select a random sample of 6 employees from each plant and administer the quality awareness examination.

8. To be classified as an ____________________, NCP would have had to be able to randomly select 18 employees and then assign the plants to each employee in a random fashion.

😍 **EXERCISES 13.2**: 1, 4, 7, 8, 10

13.3 Multiple Comparison Procedures

1. When we use analysis of variance to test whether the means of k populations are equal, ___________ of the null hypothesis allows us to conclude only that the population means are not all equal.

2. In some cases we will want to go a step further and determine where the differences among means occur.

3. To show how ____________________ procedures can be used to conduct statistical comparisons between pairs of population means.

Fisher's LSD

1. Suppose that analysis of variance provides statistical evidence to _________ the null hypothesis of equal population means. Fisher's ______________________________

procedure can be used to determine where the differences occur.

2. $\boxed{\text{Example}}$ (Section 13.1) The Chemitech experiment. Using analysis of variance, we concluded that the mean number of units produced per week are not the same for the three assembly methods. In this case, the followup question is: We believe the assembly methods differ, but where do the differences occur?

3. **Fisher's LSD Procedure**

$$H_0 : \underline{\hspace{3cm}}, \quad H_a : \mu_i \neq \mu_j$$

Test Statistic

$$t = \tag{13.16}$$

Rejection Rule ____________________

- p-value approach: RejectH_0 if p-value $\leq \alpha$.
- Critical value approach: Reject H_0 if ____________ or ____________.

where the value of $t_{\alpha/2}$ is based on a t distribution with $n_T - k$ degrees of freedom.

✎ Question .... (p616)

For the Chemitech experiment, apply Fisher's LSD Procedure to determine whether there is a significant difference between the means of population 1 (Method A) and population 2 (Method B) at the $\alpha = 0.05$ level of significance.

sol:

Area in Upper Tail	0.20	0.10	0.05	0.025	0.01	0.005
t Value (12 *df*)	0.873	1.356	1.782	2.179	2.681	3.055

4. Many practitioners find it easier to determine how large the difference between the sample means must be to reject H_0. In this case the test statistic is ___________, and the test is conducted by the following procedure.

5. **Fisher's LSD Procedure Based on the Test Statistic $\bar{x}_i - \bar{x}_j$**

$$H_0 : \mu_i = \mu_j, \quad H_a : \mu_i \neq \mu_j$$

Test Statistic

____

Rejection Rule at a Level of Significance α Reject H_0 if $|\bar{x}_i - \bar{x}_j| \geq LSD$ where

$$LSD = \qquad\qquad\qquad \tag{13.17}$$

____

6. **Confidence Interval Estimate of the Difference Between Two Population Means Using Fisher's LSD Procedure**

$$\underline{\qquad\qquad\qquad\qquad} \tag{13.18}$$

where

$$LSD = \qquad\qquad\qquad \tag{13.19}$$

____

and $t_{\alpha/2}$ is based on a t distribution with $n_T - k$ degrees of freedom.

7. If the confidence interval in expression (13.18) includes the value ________, we cannot reject the hypothesis that the two population means are equal.

8. However, if the confidence in terval does not include the value zero, we conclude that there is a difference between the population means.

✎ Question .... (p617)

For the Chemitech experiment, apply Fisher's LSD Procedure based on the Test Statistic $\bar{x}_i - \bar{x}_j$ to determine whether there is a significant difference (a) between the means of population 1 (Method A) and population 3 (Method C), (b) between the means of population 2 (Method B) and population 3 (Method C) at the $\alpha = 0.05$ level of significance. Find a 95% confidence interval estimate of the difference between the means of populations 1 and 2 and make a conclusion.

sol:

Type I Error Rates

1. ANOVA gave us statistical evidence to reject or not reject the null hypothesis of equal population means.

2. We showed how Fisher's LSD procedure can be used in such cases to determine where the differences occur. Technically, it is referred to as a ____________ or ____________ LSD test because it is employed only if we first find a significant F value by using analysis of variance.

3. To see why this distinction is important in multiple comparison tests, we need to explain the difference between a ________________ Type I error rate and an ________________ Type I error rate.

4. In the Chemitech experiment we used Fisher's LSD procedure to make three pairwise comparisons.

Test 1	Test 2	Test 3
$H_0: \mu_1 = \mu_2$	$H_0: \mu_1 = \mu_3$	$H_0: \mu_2 = \mu_3$
$H_a: \mu_1 \pm \mu_2$	$H_a: \mu_1 \pm \mu_3$	$H_a: \mu_2 \neq \mu_3$

5. In each case, we used a level of significance of $\alpha = 0.05$.

6. Therefore, ______________, if the null hypothesis is true, the probability that we will make a Type I error is $\alpha = 0.05$; hence, the probability that we will not make a Type I error on each test is _______________.

7. In discussing multiple comparison procedures we refer to this probability of a Type I error ($\alpha = 0.05$) as the ______________________________; It indicate the level of significance associated with a ________ pairwise comparison.

8. What is the probability that in making three pairwise comparisons, we will commit a Type I error on ____________ of the three tests?

   (a) To answer this question, note that the probability that we will not make a Type I error on any of the three tests is _________________________.

   (b) The probability of making at least one Type I error is ___________________.

   (c) Thus, when we use Fisher's LSD procedure to make all three pairwise comparisons, the Type I error rate associated with this approach is not 0.05, but actually 0.1426; we refer to this error rate as the _________ or ________________ Type I error rate.

9. To avoid confusion, we denote the experimentwise Type I error rate as ________.

10. The experimentwise Type I error rate gets larger for problems with more populations. For example, a problem with five populations has 10 possible pairwise comparisons. If we tested all possible pairwise comparisons by using Fisher's LSD with a comparisonwise error rate of $\alpha = 0.05$, the experimentwise Type I error rate would be ____________________.

11. In such cases, practitioners look to alternatives that provide better control over the experimentwise error rate.

12. One alternative for controlling the overall experimentwise error rate, referred to as the ______________________, involves using a smaller comparisonwise error rate for each test.

13. For example, if we want to test C pairwise comparisons and want the maximum probability of making a Type I error for the overall experiment to be α_{EW}, we simply use a comparisonwise error rate equal to __________.

14. In the Chemitech experiment, if we want to use Fisher's LSD procedure to test all three pairwise comparisons with a maximum experimentwise error rate of _____________, we set the comparisonwise error rate to be ___________________.

15. (Recall Chapter 9) For a fixed sample size, any decrease in the probability of making a Type I error will result in an increase in the probability of making a __________ error, which corresponds to accepting the hypothesis that the two population means are equal when in fact they are not equal.

16. As a result, many practitioners are reluctant to perform individual tests with a low comparisonwise Type I error rate because of the increased risk of making a Type II error.

17. Several other procedures, such as __________________ and __________________ ____________, have been developed to help in such situations. However, there is considerable controversy in the statistical community as to which procedure is "best." The truth is that no one procedure is best for all types of problems.

😍 **EXERCISES 13.3**: 13, 15, 18, 19

13.4 Randomized Block Design*

13.5 Factorial Experiment*

😍 **SUPPLEMENTARY EXERCISES**: 35, 37