Control Charts for Multivariate Processes

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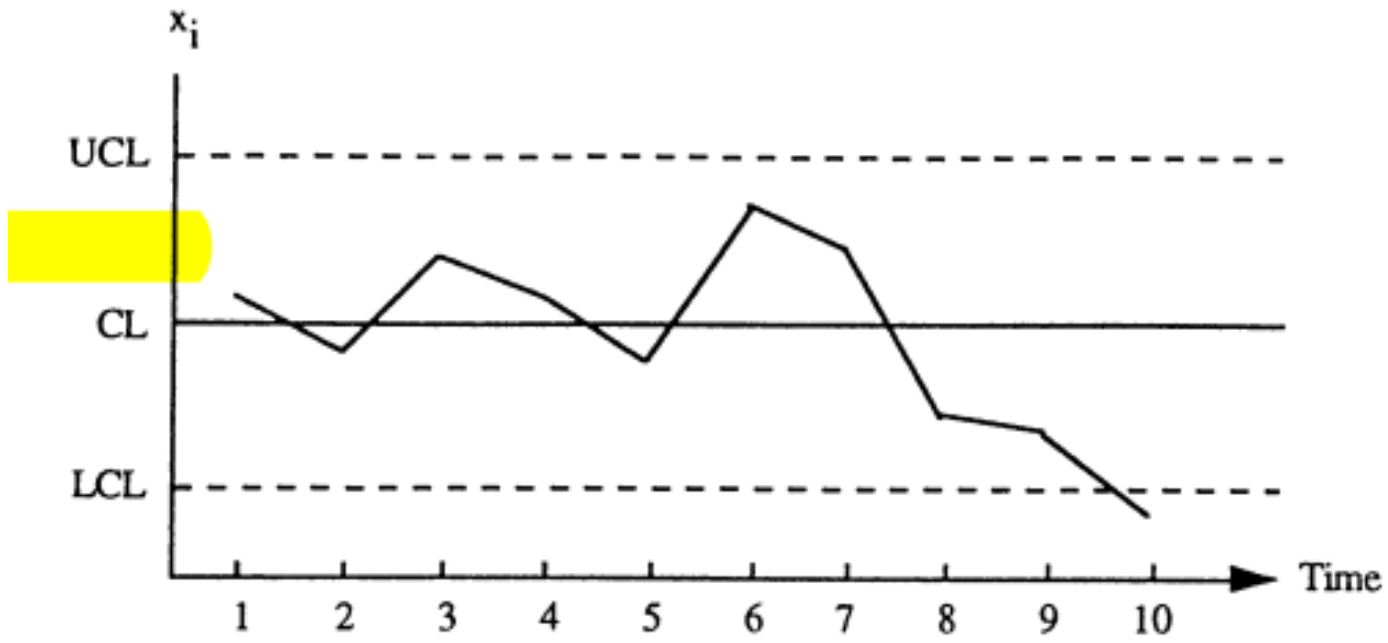
Introduction and Background

- 一般常見之管制圖:X Chart ,$\bar{X}$ $Chart$, CUSUM Chart
- 一般管制圖之限制: 單變數的、假設為常態分配(較不切實際)
- 在現實生活中，會有多元的品質變數。
- 以藥片 為例: 重量、硬度、厚度、長寬高(彼此相關的)
- 有些建立多元變數管制圖的方法有被提出，但還是需要假設為常態分配且很難被視覺化和解釋。

X Chart



$UCL=CL+z\alpha_{/2} * \sigma$

$CL=\mu$

$LCL=CL+z\alpha_{/2} * \sigma$

Data depth

- 將多元的變數轉為一元的指標
- 不需對原本的數據有任何假設
- 較好視覺化和解釋

Mahalanobis distance

- G、F~任意分配且G和F為不同分配
- 從G中隨機抽m個樣本稱為$Y_1, \dots, Y_m$
- 從F中隨機抽n個樣本稱為$X_1, \dots, X_n$
- 把每一個Y_i 和X_i都轉為Mahalanobis distance.
- $D_{G_m}(Y_i) = \frac{1}{[1+(Y_i-\bar{Y})'S^{-1}(Y_i-\bar{Y})]}$, $\bar{Y}$ is sample mean of $Y_1, \dots, Y_m$ and S is the sample covariance matrix.
- $D_{G_m}(X_i) = \frac{1}{[1+(X_i-\bar{Y})'S^{-1}(X_i-\bar{Y})]}$

Mahalanobis distance

- 再把$D_{G_m}(X_i)$做評級(rank)
- $r_{G_m}(X_i) = \#\frac{\{Y_j | D_{G_m}(Y_j) \leq D_{G_m}(X_i), j=1,\ldots,m\}}{m}$

- $r_{G_m}(X_i)$之值越小，說明有越小比例的Y_i比X_i還小，代表該X_i值越可能為異常值。

SIMULATION RESULTS

- Let G~N($\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$) ,F~N($\mu = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$
- 從G生成540筆資料，把前500筆命名為$Y_1, \dots, Y_{500}$，後40筆命名為$X_1, \dots, X_{40}$
- 再從F生成40筆資料，把它名為$X_{41}, \dots$
- 接著把$Y_1, \dots, Y_{500}$和$X_1, \dots, X_{80}$都轉為Mahalanobis distance
- 可得到$D_{G_m}(Y_i), i = 1\sim500$, $D_{G_m}(X_i), \mathrm{i} = 1\sim80$
- 最後再把$D_{G_m}(X_i), \mathrm{i} = 1\sim80$做評級，得到$r_{G_m}(X_i), \mathrm{i} = 1\sim80$

X	D(X)	r(X)	X	D(X)	r(X)	X	D(X)	r(X)	X	D(X)	r(X)
1	.0028	.082	21	.0711	.502	41	0	.022	61	.0932	.588
2	.2263	.948	22	.0645	.472	42	0	.022	62	0	.022
3	.1794	.840	23	.0103	.186	43	0	.022	63	0	.022
4	.0196	.256	24	.0797	.542	44	.0107	.194	64	0	.022
5	.1144	.670	25	.0870	.566	45	0	.022	65	0	.022
6	.0025	.074	26	.0051	.114	46	.0041	.100	66	0	.022
7	.0115	.196	27	.0518	.424	47	0	.022	67	0	.022
8	.0443	.392	28	0	.022	48	0	.022	68	.0123	.202
9	.0389	.358	29	.0044	.102	49	.0111	.194	69	0	.022
10	.0268	.296	30	.0903	.576	50	.0261	.290	70	.1984	.896
11	0	.022	31	.1900	.866	51	0	.022	71	.0250	.280
12	.1962	.888	32	.1621	.800	52	0	.022	72	.0087	.160
13	.1651	.812	33	.1499	.768	53	0	.022	73	0	.022
14	.1835	.852	34	.0757	.528	54	0	.022	74	0	.022
15	.0249	.280	35	.0514	.420	55	0	.022	75	0	.022
16	.0583	.446	36	.0581	.444	56	0	.022	76	0	.022
17	.1106	.658	37	.1096	.656	57	0	.022	77	0	.022
18	.0022	.068	38	.0570	.436	58	0	.022	78	0	.022
19	.2315	.962	39	.2082	.920	59	0	.022	79	0	.022
20	.0366	.348	40	.1927	.876	60	0	.022	80	0	.022

建立Data depth 之管制圖—r chart

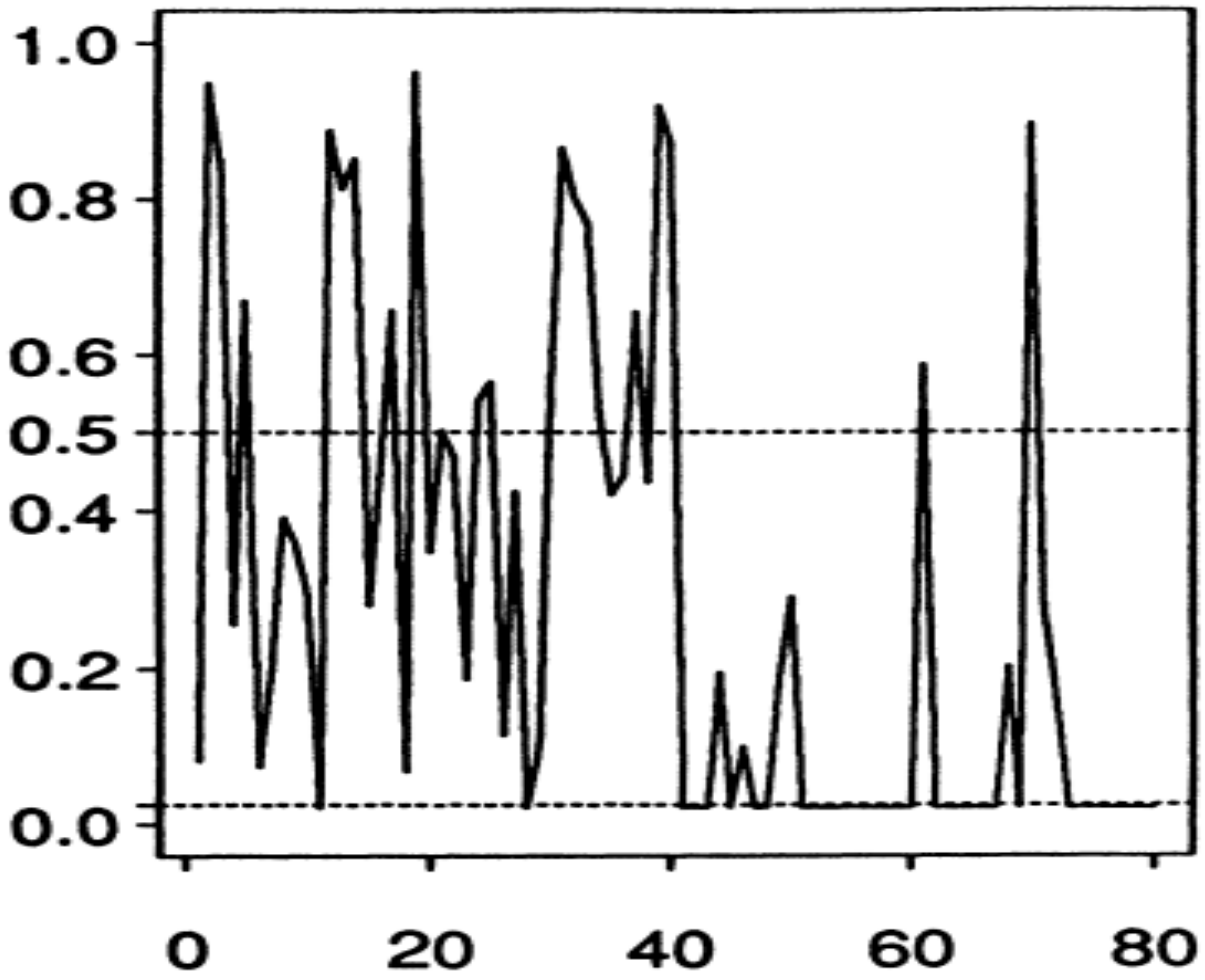


Figure 1. r Chart.

CL=0.5

UCL=0.025

建立Data depth 之管制圖—Q chart(n=4)

- $Qr_{G_m}(X_i) = \frac{1}{n}\sum_{i=1}^{n} r_{G_m}(X_i)$

Table 2. Q-values (n = 4)

.5315	.3330	.3910	.5975	.5090	.4255	.2815	.5860	.5400	.7220
.0650	.0415	.1320	.0220	.0220	.1635	.0670	.3395	.0220	.0220

建立Data depth 之管制圖—Q chart(n=4)

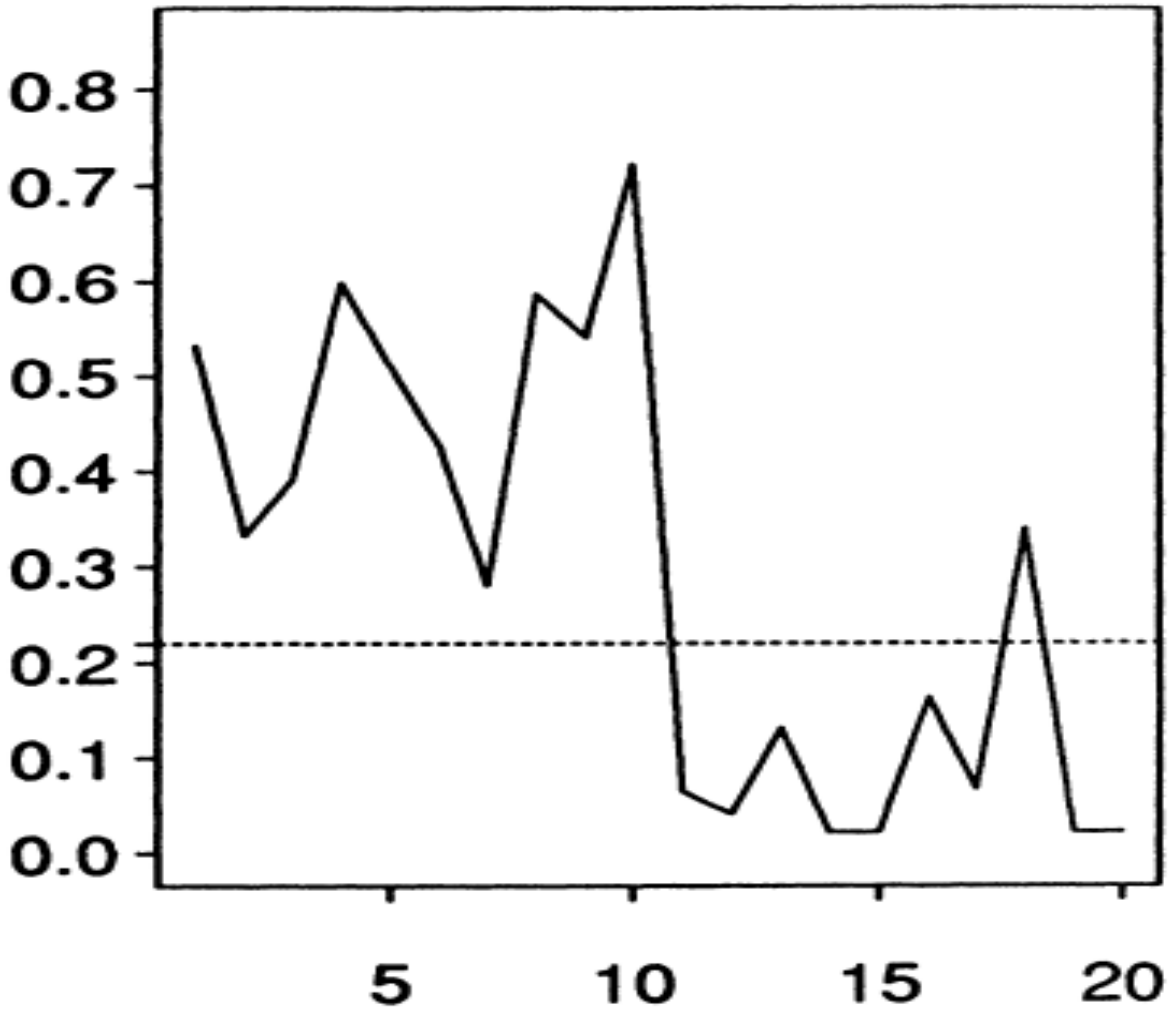


Figure 2. Q Chart (n = 4).

CL=0.5

UCL=0.220

Conclusion

- 此方法也可以用在更為複雜的管制圖，像是EWMA chart , moving average chart …
- 除了把資料轉為Mahalanobis distance外，也可以轉為其他種類之深度，像是majority depth, simplicial depth, and Tukey‘s depth …