

A new nonparametric EWMA Sign Control Chart

Su-Fen Yang , Jheng-Sian Lin , Smiley W. Cheng
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統碩一 112354023 溫怡茹

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Introduction

- ▶ In practice,
the in-control process mean may not be the process target
⇒ monitor the deviation from the process target
- ▶ if we had no knowledge of the underlying population distribution?
⇒ Shewhart variables control charts are not suitable
⇒ need nonparametric approaches
- ▶ Shewhart chart is its inability to detect small shifts
⇒ EWMA chart

Introduction

- ▶ In this paper,
propose a **new EWMA Sign Chart** for variables data to monitor the deviation from the process target

The EWMA Sign Chart

▶ assume quality characteristic X has a target value T

▶ $Y = X - T$

process proportion : $p = P(Y > 0)$

if process in control $\Rightarrow p = 0.5$

out of control $\Rightarrow p = p_1 \neq 0.5$

▶ X_1, X_2, \dots, X_n

$Y_j = X_j - T$ and $I_j = \begin{cases} 1, & \text{if } Y_j > 0, \\ 0, & \text{otherwise,} \end{cases}$ for $j = 1, 2, \dots, n$.

$M = \sum_{j=1}^n I_j \sim \text{Bin}(n, 0.5)$

The EWMA Sign Chart

▶ $EWMA_{M_i} = \lambda M_i + (1 - \lambda)EWMA_{M_{i-1}}, 0 < \lambda \leq 1$

▶ $EWMA_{M_0} = n/2$

$$E(EWMA_{M_i}) = n/2$$

$$Var(EWMA_{M_i}) = \frac{\lambda[1-(1-\lambda)^{2i}]}{2-\lambda} (1/4n)$$

if time is infinite $\Rightarrow Var(EWMA_{M_i}) = \frac{\lambda}{2-\lambda} (1/4n)$

The EWMA Sign Chart

- ▶ construct the EWMA Sign Chart :

$$UCL_{EWMA_M} = n/2 + k \sqrt{\frac{\lambda}{2-\lambda} (1/4n)}$$

$$CL_{EWMA_M} = n/2$$

$$LCL_{EWMA_M} = n/2 - k \sqrt{\frac{\lambda}{2-\lambda} (1/4n)}$$

Design a chart (n, k, λ) with $ARL_0 \approx 370$

Table 1

The k values with various combinations of (n, λ) under $ARL_0 \approx 370$.

λ	$ARL_0 \approx 370$												
	n	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9
9		2.48	2.70	2.77	2.83	2.85	2.85	2.88	2.87	2.86	2.83	2.85	2.94
10		2.49	2.69	2.77	2.84	2.86	2.86	2.89	2.88	2.86	2.85	2.81	2.70
11		2.49	2.70	2.78	2.84	2.88	2.88	2.89	2.89	2.88	2.86	2.85	2.79
12		2.49	2.68	2.79	2.84	2.86	2.89	2.90	2.90	2.90	2.89	2.89	2.91
13		2.50	2.69	2.78	2.84	2.87	2.89	2.90	2.91	2.90	2.90	2.88	2.95
14		2.49	2.69	2.80	2.85	2.86	2.89	2.91	2.92	2.91	2.89	2.88	2.81
15		2.49	2.69	2.78	2.84	2.87	2.90	2.91	2.92	2.91	2.91	2.91	2.89
16		2.49	2.68	2.79	2.84	2.86	2.91	2.92	2.92	2.91	2.91	2.91	2.96
17		2.48	2.70	2.79	2.85	2.87	2.89	2.93	2.92	2.93	2.92	2.91	2.83
18		2.50	2.70	2.79	2.84	2.86	2.90	2.92	2.95	2.92	2.92	2.92	2.89
19		2.48	2.70	2.79	2.86	2.87	2.89	2.92	2.93	2.93	2.93	2.92	2.96
20		2.47	2.71	2.77	2.84	2.89	2.89	2.92	2.93	2.93	2.92	2.92	2.89
21		2.50	2.67	2.79	2.85	2.88	2.90	2.93	2.92	2.93	2.93	2.93	2.90
22		2.49	2.70	2.79	2.84	2.88	2.90	2.93	2.93	2.94	2.93	2.94	2.96
23		2.49	2.70	2.79	2.84	2.88	2.90	2.93	2.94	2.94	2.93	2.93	2.90
24		2.49	2.70	2.78	2.85	2.87	2.91	2.93	2.94	2.94	2.94	2.94	2.92
25		2.49	2.69	2.80	2.85	2.88	2.90	2.93	2.95	2.94	2.94	2.94	2.97

- ▶ k values are very close for $n = 9(1)25$ under the specified λ

EWMA Sign Control Chart limits

for $k = 2.49, \lambda = 0.05, ARL_0 \approx 370$

Table 2

EWMA Sign Control Chart limits for $k = 2.49, \lambda = 0.05, ARL_0 \approx 370$.

<i>n</i>	<i>CL</i>	<i>LCL</i>	<i>UCL</i>	<i>n</i>	<i>CL</i>	<i>LCL</i>	<i>UCL</i>	<i>n</i>	<i>CL</i>	<i>LCL</i>	<i>UCL</i>
9	4.5	3.90	5.10	15	7.5	6.73	8.27	21	10.5	9.59	11.41
10	5	4.37	5.63	16	8	7.20	8.80	22	11	10.06	11.94
11	5.5	4.84	6.16	17	8.5	7.68	9.32	23	11.5	10.54	12.46
12	6	5.31	6.69	18	9	8.15	9.85	24	12	11.02	12.98
13	6.5	5.78	7.22	19	9.5	8.63	10.37	25	12.5	11.50	13.50
14	7	6.25	7.75	20	10	9.11	10.89				

The ARL_1 of the EWMA Sign Chart

Table 3

The ARL_1 values under $\lambda = 0.05$ and $k = 2.49$.

n	p																		
	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
9	4	4	5	5	7	9	12	21	57	384	57	21	12	9	7	5	5	4	4
10	3	4	4	5	6	8	11	19	52	371	52	19	11	8	6	5	4	4	3
11	3	4	4	5	6	8	11	18	48	370	48	18	11	8	6	5	4	4	3
12	3	4	4	5	6	7	10	17	46	380	45	17	10	7	6	5	4	4	3
13	3	3	4	5	6	7	10	16	43	377	43	16	10	7	6	5	4	3	3
14	3	3	4	4	5	7	9	15	41	378	40	15	9	7	5	4	4	3	3
15	3	3	4	4	5	7	9	15	39	386	39	15	9	7	5	4	4	3	3
16	3	3	3	4	5	6	9	14	37	371	36	14	9	6	5	4	3	3	3
17	3	3	3	4	5	6	8	14	35	384	35	14	8	6	5	4	3	3	3
18	3	3	3	4	5	6	8	13	34	375	34	13	8	6	5	4	3	3	3
19	3	3	3	4	5	6	8	13	33	388	33	13	8	6	5	4	3	3	3
20	3	3	3	4	4	6	8	12	32	389	32	12	8	6	4	4	3	3	3
21	2	3	3	4	4	5	7	12	30	379	30	12	7	5	4	4	3	3	2
22	2	3	3	4	4	5	7	12	29	383	29	12	7	5	4	4	3	3	2
23	2	3	3	4	4	5	7	11	28	383	28	11	7	5	4	3	3	3	2
24	2	3	3	4	4	5	7	11	28	381	27	11	7	5	4	3	3	3	2
25	2	3	3	4	4	5	7	11	27	377	27	11	7	5	4	3	3	3	2

Close to 370

- ▶ ARL_1 inversely related to n and $|p_1 - 0.5|$

Example

Example

- ▶ fill volume of soft – drink beverage bottles

Sample	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	M	$EWMA_M$
1	2.5	0.5	2	-1	1	-1	0.5	1.5	0.5	-1.5	7	5.10
2	0	0	0.5	1	1.5	1	-1	1	1.5	-1	6	5.15
3	1.5	1	1	-1	0	-1.5	-1	-1	1	-1	4	5.09
4	0	0.5	-2	0	-1	1.5	-1.5	0	-2	-1.5	2	4.93
5	0	0	0	-0.5	0.5	1	-0.5	-0.5	0	0	2	4.79
6	1	-0.5	0	0	0	0.5	-1	1	-2	1	4	4.75
7	1	-1	-1	-1	0	1.5	0	1	0	0	3	4.66
8	0	-1.5	-0.5	1.5	0	0	0	-1	0.5	-0.5	2	4.53
9	-2	-1.5	1.5	1.5	0	0	0.5	1	0	1	5	4.55
10	-0.5	3.5	0	-1	-1.5	-1.5	-1	-1	1	0.5	3	4.47
11	0	1.5	0	0	2	-1.5	0.5	-0.5	2	-1	4	4.45
12	0	-2	-0.5	0	-0.5	2	1.5	0	0.5	-1	3	4.38
13	-1	-0.5	-0.5	-1	0	0.5	0.5	-1.5	-1	-1	2	4.26
14	0.5	1	-1	-0.5	-2	-1	-1.5	0	1.5	1.5	4	4.25
15	1	0	1.5	1.5	1	-1	0	1	-2	-1.5	5	4.24

$M = \text{Sum of positive differences } (X_i - 0), i = 1, 2, \dots, 15.$

Example

Table 2

EWMA Sign Control Chart limits for $k = 2.49$, $\lambda = 0.05$, $ARL_0 \approx 370$.

n	CL	LCL	UCL	n	CL	LCL	UCL	n	CL	LCL	UCL
9	4.5	3.90	5.10	15	7.5	6.73	8.27	21	10.5	9.59	11.41
10	5	4.37	5.63	16	8	7.20	8.80	22	11	10.06	11.94
11	5.5	4.84	6.16	17	8.5	7.68	9.32	23	11.5	10.54	12.46
12	6	5.31	6.69	18	9	8.15	9.85	24	12	11.02	12.98
13	6.5	5.78	7.22	19	9.5	8.63	10.37	25	12.5	11.50	13.50
14	7	6.25	7.75	20	10	9.11	10.89				

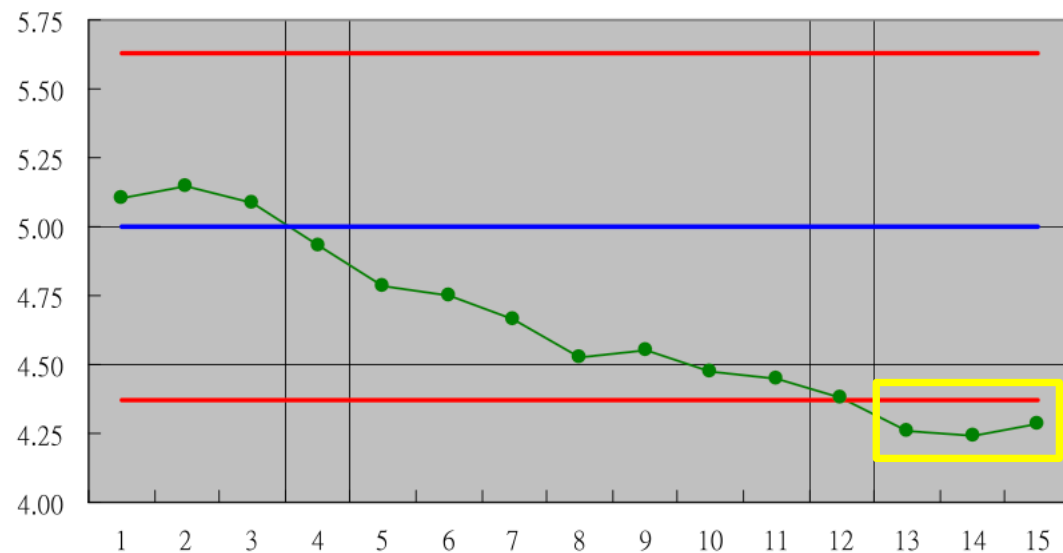


Fig. 2. EWMA Sign Chart.

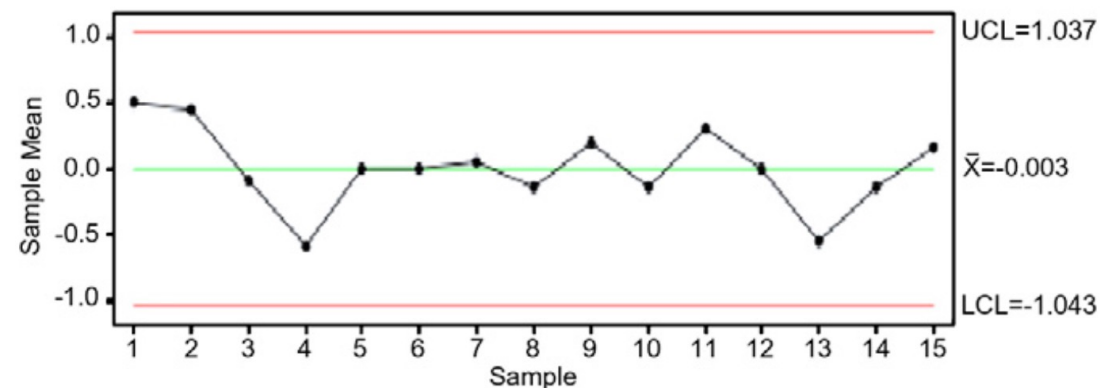


Fig. 3. X-bar chart.

► requires normality

Arcsine EWMA Sign Chart

Arcsine EWMA Sign Chart

- ▶ Apply the arcsine transformation :

$$Y = \sin^{-1}\left(\sqrt{\frac{M}{n}}\right) \sim N\left(\sin^{-1}(\sqrt{p}), \frac{1}{4n}\right)$$

- ▶ $EWMA_{Y_i} = \lambda Y_i + (1 - \lambda)EWMA_{Y_{i-1}}, 0 < \lambda \leq 1$

$$EWMA_{Y_0} = \sin^{-1} \sqrt{0.5}$$

$$E(EWMA_{Y_i}) = \sin^{-1} \sqrt{0.5}$$

$$Var(EWMA_{M_i}) = \frac{\lambda[1-(1-\lambda)^{2i}]}{2-\lambda} (1/4n)$$

Arcsine EWMA Sign Chart

- ▶ construct the Arcsine EWMA Sign Chart :

$$UCL_{EWMA_Y} = \sin^{-1} \sqrt{0.5} + k \sqrt{\frac{\lambda}{2-\lambda} (1/4n)}$$

$$CL_{EWMA_Y} = \sin^{-1} \sqrt{0.5}$$

$$LCL_{EWMA_Y} = \sin^{-1} \sqrt{0.5} - k \sqrt{\frac{\lambda}{2-\lambda} (1/4n)}$$

The Arcsine EWMA_γ Sign Chart (Example)

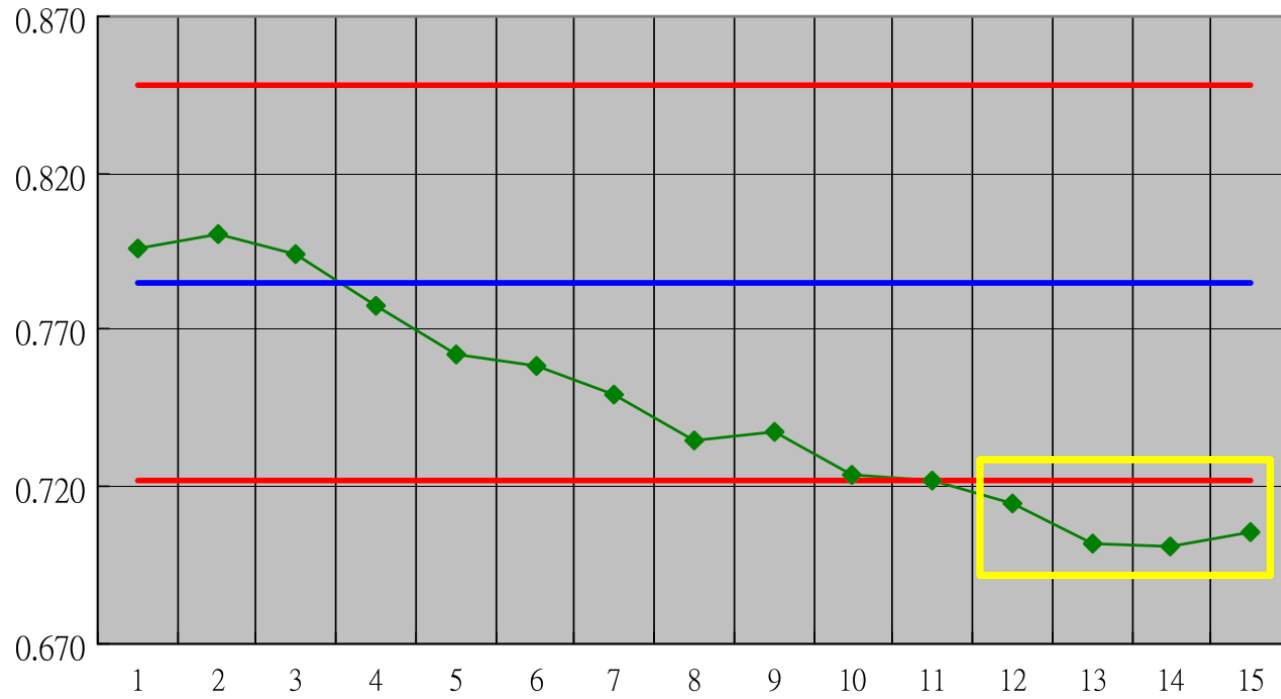


Fig. 4. The Arcsine EWMA_γ Sign Chart.

The Arcsine EWMA_Y Sign Chart

Table 4

The ARL values of Arcsine EWMA_Y Sign Chart ($\lambda = 0.05$, $k = 2.49$).

n	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
9	3	4	4	5	6	8	12	20	56	370	56	20	12	8	6	5	4	4	3
10	3	3	4	5	6	8	11	19	52	370	52	19	11	8	6	5	4	3	3
11	3	3	4	5	6	8	11	18	48	370	48	18	11	8	6	5	4	3	3
12	3	3	4	4	6	7	10	17	45	370	45	17	10	7	6	4	4	3	3
13	3	3	4	4	5	7	10	16	42	370	42	16	10	7	5	4	4	3	3
14	2	3	3	4	5	7	9	15	40	370	40	15	9	7	5	4	3	3	2
15	2	3	3	4	5	6	9	15	38	370	38	15	9	6	5	4	3	3	2
16	2	3	3	4	5	6	9	14	36	370	36	14	9	6	5	4	3	3	2
17	2	3	3	4	5	6	8	13	35	370	35	13	8	6	5	4	3	3	2
18	2	3	3	4	4	6	8	13	33	370	33	13	8	6	4	4	3	3	2
19	2	3	3	4	4	6	8	13	32	370	32	13	8	6	4	4	3	3	2
20	2	2	3	3	4	5	8	12	31	370	31	12	8	5	4	3	3	2	2
21	2	2	3	3	4	5	7	12	30	370	30	12	7	5	4	3	3	2	2
22	2	2	3	3	4	5	7	12	29	370	29	12	7	5	4	3	3	2	2
23	2	2	3	3	4	5	7	11	28	370	28	11	7	5	4	3	3	2	2
24	2	2	3	3	4	5	7	11	27	370	27	11	7	5	4	3	3	2	2
25	2	2	3	3	4	5	7	11	26	370	26	11	7	5	4	3	3	2	2

Conclusion

Proposed a new nonparametric EWMA Sign Chart

- ▶ monitor the deviation from process target
- ▶ when the underlying distribution is unknown or nonnormal
- ▶ recommend **the Arcsine EWMA Sign Chart**,
if we are concerned with attaining the proper ARL values

Thank You